Fractional Viscoelasticity of Soft Elastomers and Auxetic Foams

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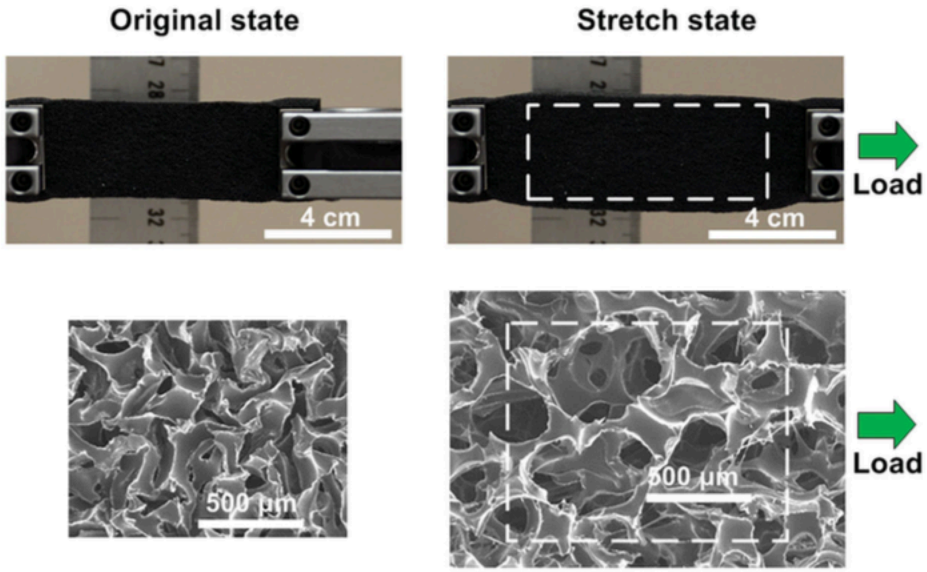
Motivation

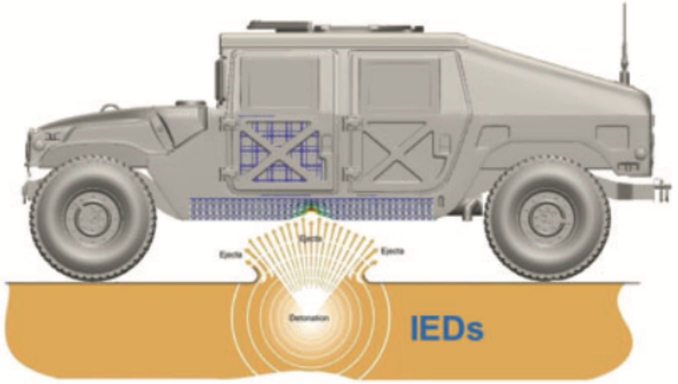
- Dielectric elastomers:
  - Robotics
  - Energy harvesting
  - Optical switches

Figure: Digital photographs and corresponding SEM images of auxetic foam before (left) and after (right) stretching force was applied in transverse direction with 40% strain.[1]



- Auxetic foams - negative Poisson's ratio:
  - Indentation resistance
  - Optimal dynamics
  - Acoustic properties of wave absorbers

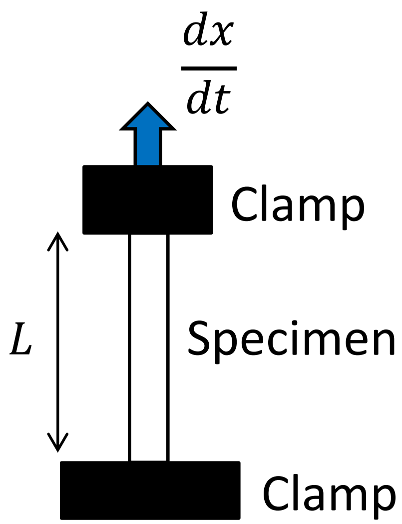
Figure: Application of auxetic panels to improve impact resistance capabilities of armored vehicles.[2]



1. Li, Yan, et al. "Poisson Ratio and Piezoresistive Sensing: A New Route to High-Performance 3D Flexible and Stretchable Sensors of Multimodal Sensing Capability." *Advanced Functional Materials* 26.17 (2016): 2900-2908. Figure 1(b).
2. Imbalzano, Gabriele, et al. "Three-dimensional modelling of auxetic sandwich panels for localised impact resistance." *Journal of Sandwich Structures & Materials* 19.3 (2017): 291-316. Figure 1.

Experimental Setup

- Materials:
  - Very High Bond (VHB) 4910 & 4949
  - Auxetic Foam
- Specimens cycled
- Stretch rate: $\frac{d\lambda}{dt} = \frac{1}{L_0}\frac{dx}{dt}$
  - $\frac{d\lambda}{dt}$ is the stretch rate (Hz)
  - $\frac{dx}{dt}$ is the speed of the moving clamp head (mm/s)
  - L_0 is the initial length of the VHB specimen (mm)



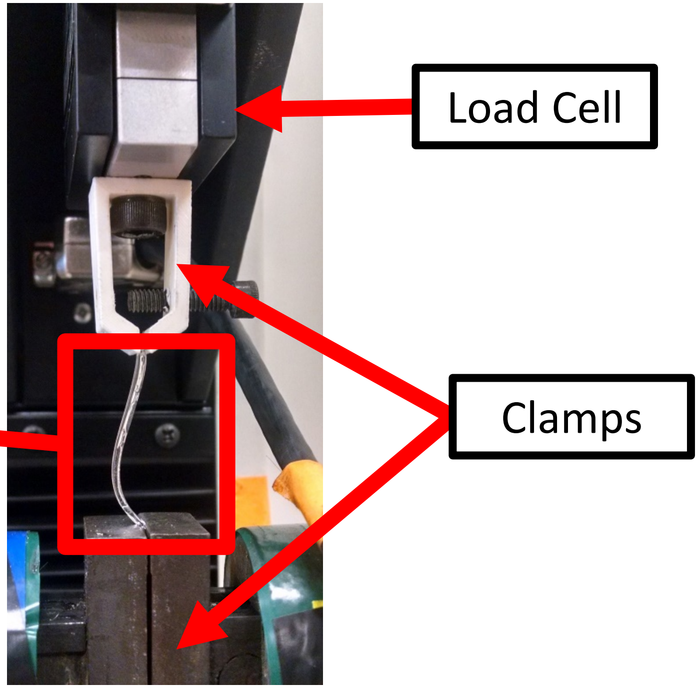
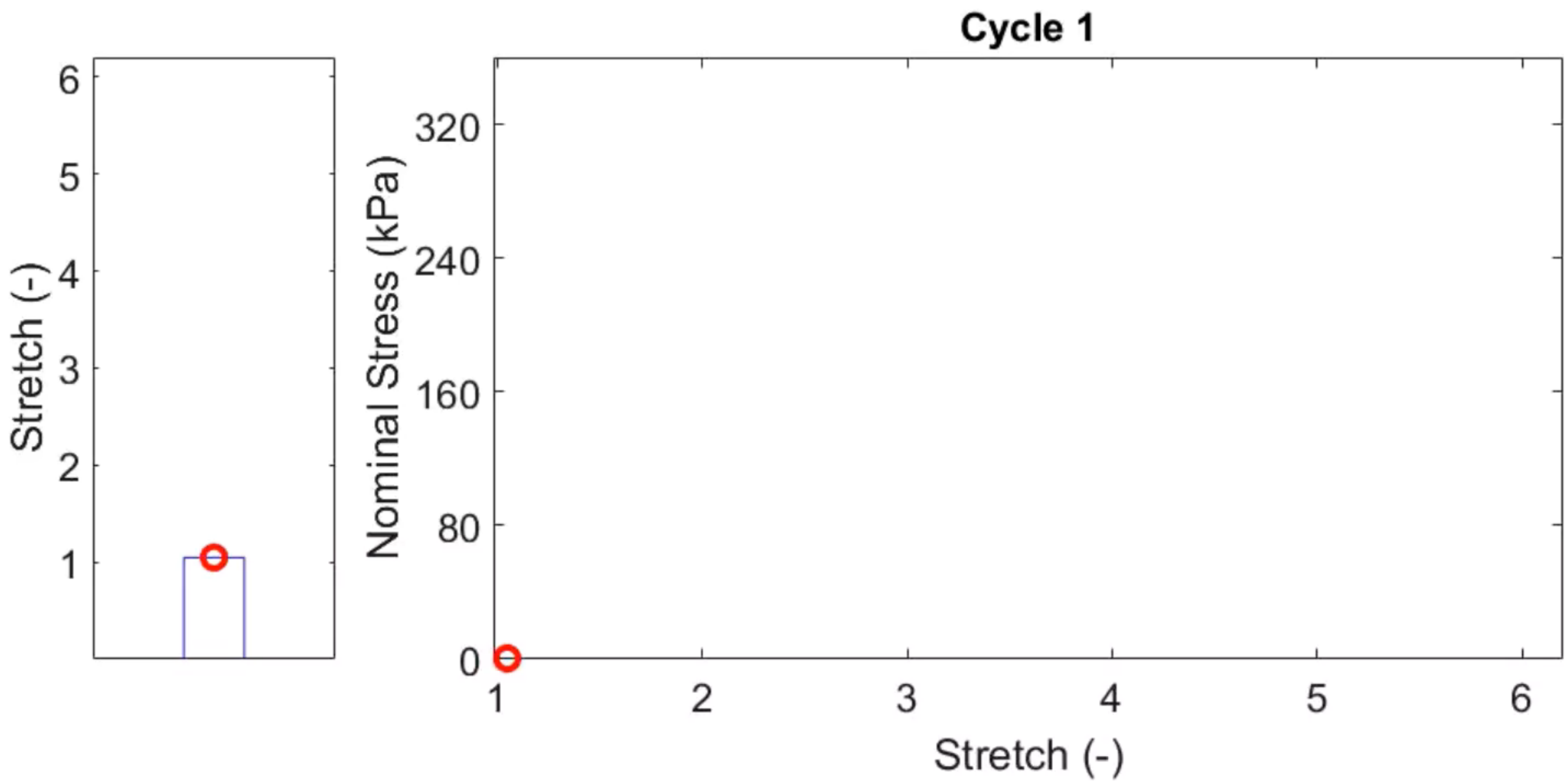


Figure: MTS tensile testing of VHB 4910

Experimental Observations: VHB 4910 Specimen



Experimental Observations

- Hysteresis
- Stress response decays
- Recovery
- Steady state

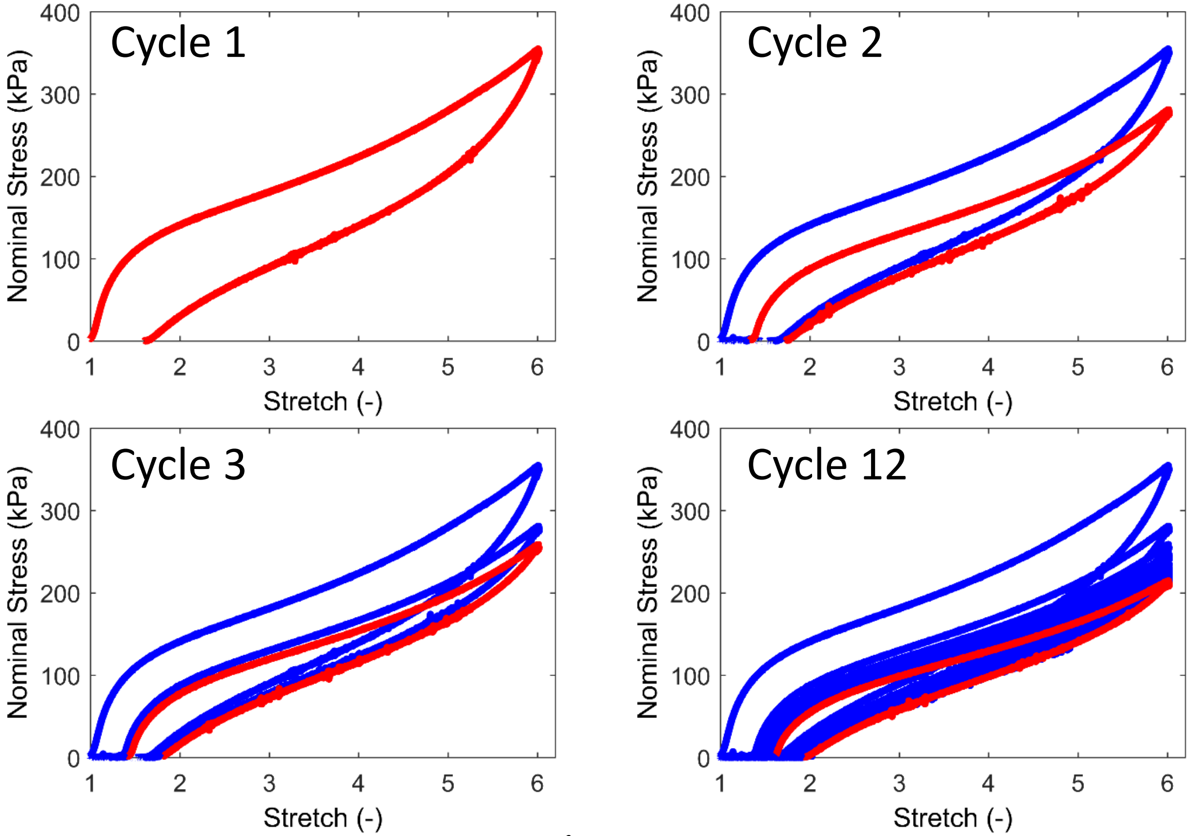


Figure: Cyclic loading of VHB 4910 at $\frac{d\lambda}{dt} = 0.67\ Hz$. Steady state hysteresis observed by the 12th cycle.

Experimental Observations: VHB 4949

- Loading history

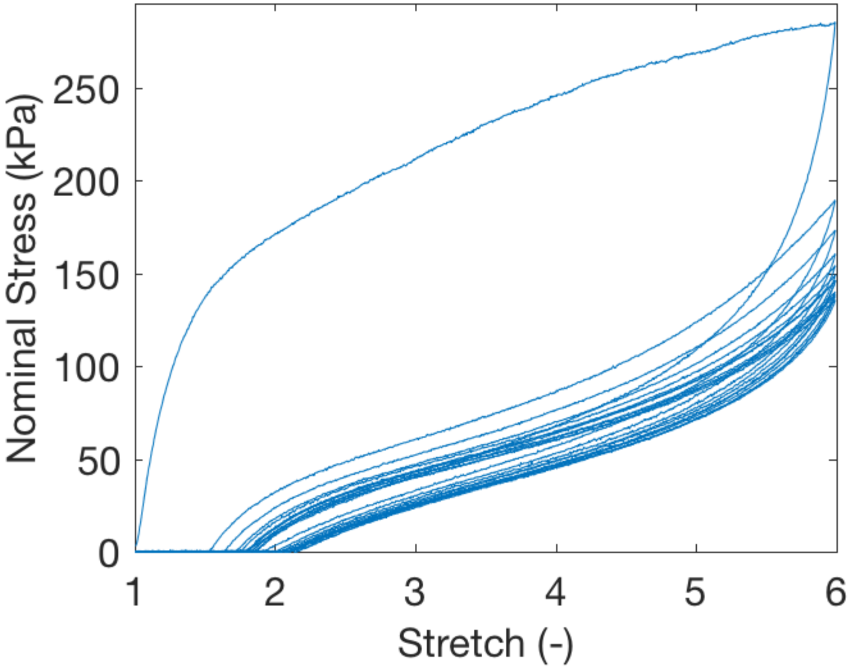


Figure: Nominal stress response for soft elastomer VHB 4949 over 12 loading/relaxation cycles.

- Rate-dependence

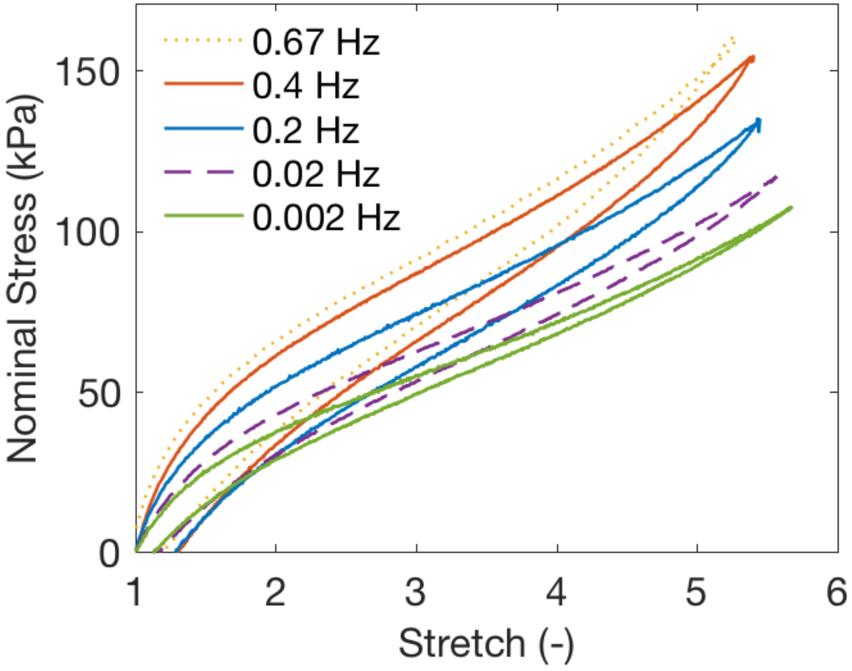


Figure: Steady state hysteresis loops for soft elastomer VHB 4949.

Experimental Observations: Auxetic Foam

- Loading history

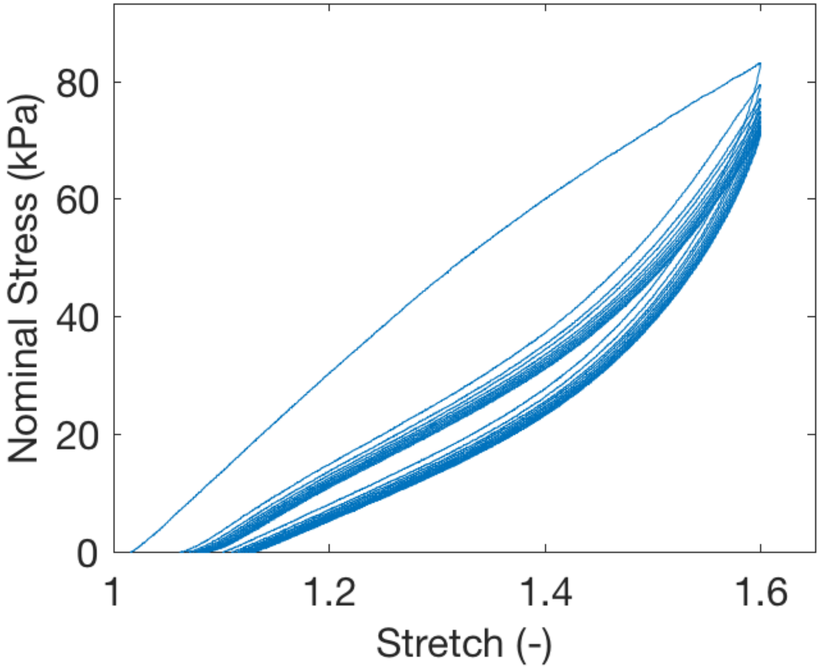


Figure: Nominal stress response for auxetic foam over 12 loading/relaxation cycles.

- Rate-dependence

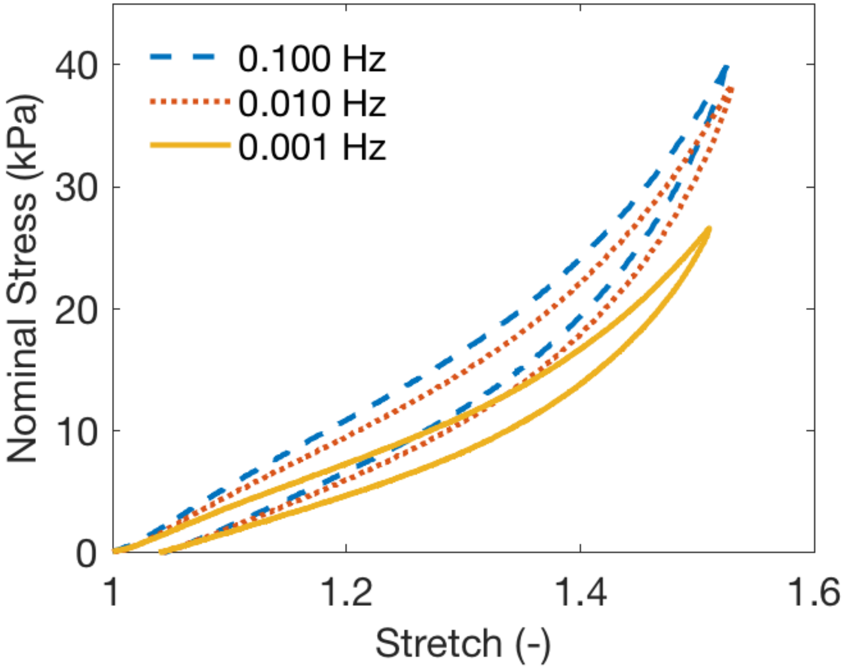


Figure: Steady state hysteresis loops for auxetic foam.

Theory:

- Total Energy Density: $\psi = \psi_\infty(F_{iK}, \Theta) + \Upsilon(F_{iK}, \Theta, \Gamma^\nu_{iK})$
  - F_{iK} - deformation gradient
  - Θ - temperature
  - Γ^ν_{iK} - set of non-conserved internal strains
- Conserved, hyperelastic energy function[1]

$$\psi_\infty = \frac{1}{6} G_c I_1 - G_c \lambda_{max}^2 \log(3\lambda_{max}^2 - I_1) + G_e \sum_j (\lambda_j + \frac{1}{\lambda_j})$$

- Non-conserved energy function:

$$\Upsilon = \sum_\nu [\frac{1}{2} \gamma_\nu (F_{iK} - \Gamma^\nu_{iK})(F_{iK} - \Gamma^\nu_{iK})]$$

- Model Parameters
  - G_c - Crosslink modulus
  - G_e - Entanglement modulus
  - λ_{max} - Maximum extension of affine tube
  - γ_ν - Proportional to viscosity of polymer network

1. Davidson, Jacob D., and N. C. Goulbourne. "A nonaffine network model for elastomers undergoing finite deformations." Journal of the Mechanics and Physics of Solids 61.8 (2013): 1784-1797.

Theory:

- Nominal Stress

$$s_{iK} = \frac{\partial \hat{\psi}}{\partial F_{iK}} = \frac{\partial \psi_\infty}{\partial F_{iK}} - pJH_{iK} + \frac{\partial \Upsilon}{\partial F_{iK}}$$

- Viscoelastic Stress

$$Q_{iK}^{\nu} = -\frac{\partial \hat{\psi}}{\partial \Gamma_{iK}^{\nu}} = -\frac{\partial \Upsilon}{\partial \Gamma_{iK}^{\nu}}$$

- Viscoelastic constitutive law:[1,2]

$$Q_{iK}^{\nu} = \eta \left(\frac{d\Gamma_{iK}^{\nu}}{dt}\right) \rightarrow Q_{iK} = \eta D_t^{\alpha} F_{iK}$$

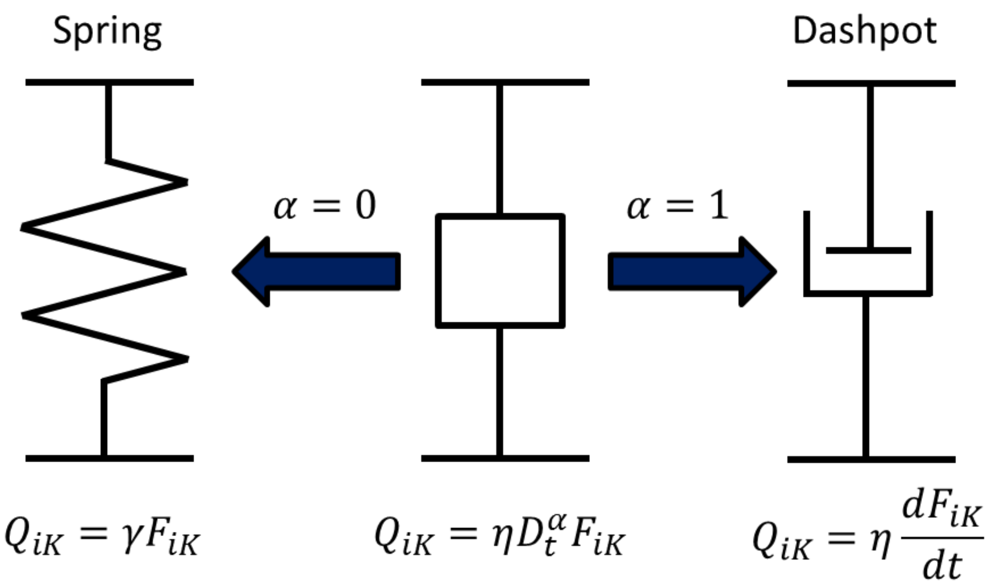


Figure: Fractional order spring-dashpot system. As the fractional order approaches 0 the system behaves like a dashpot. Likewise, as the fractional order approaches 1 the system behaves like a spring.

1. Miles, P., Hays, M., Smith, R., Oates, W., "Bayesian Uncertainty Analysis of Finite Deformation Viscoelasticity." *Mechanics of Materials*, 2015, Vol. 91, pp. 35-49.
2. Mashayekhi, S., Miles, P., Hussaini, M. Y., Oates, W., "Fractional Viscoelasticity in Fractal and Non-Fractal Media: Theory, Experimental Validation, and Uncertainty Analysis." *Journal of the Mechanics and Physics of Solids*, 2017, Vol. 111, pp. 134-156.

Uncertainty Quantification: Bayesian Statistical Analysis

- Statistical Model: $M^{data}(i) = M(i;\theta) + \varepsilon_i, \quad i = 1, \dots, N$
- Bayes' Relation

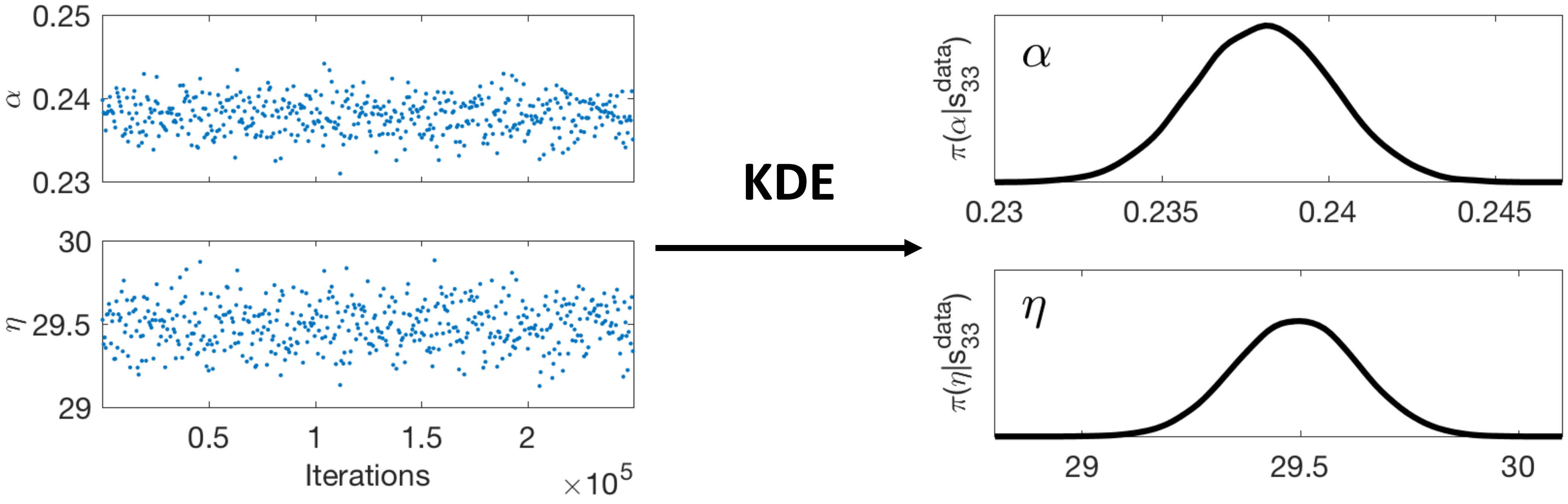
$$\pi(\theta | M^{data}) = \frac{p(M|\theta)\pi_0(\theta)}{\int_{\mathbb{R}_p} p(M|\theta)\,\pi_0(\theta)d\theta}$$

- Posterior Density: $\pi(\theta | M^{data})$
- Prior Density: $\pi_0(\theta)$
- Likelihood Function: $p(M|\theta) = e^{-\sum_{i=1}^{n}\left[M^{data}(i) - M(i;\theta)\right]^2/(2\sigma^2)}$
  - Assume observation errors are iid: $\varepsilon_i \sim N(0, \sigma^2)$.

- For this problem: $\theta = [\eta, \alpha]$

Parameter Estimation:

- Parameter chains to posterior densities, $\pi(\theta|M^{data})$
  - Kernel Density Estimator (KDE)



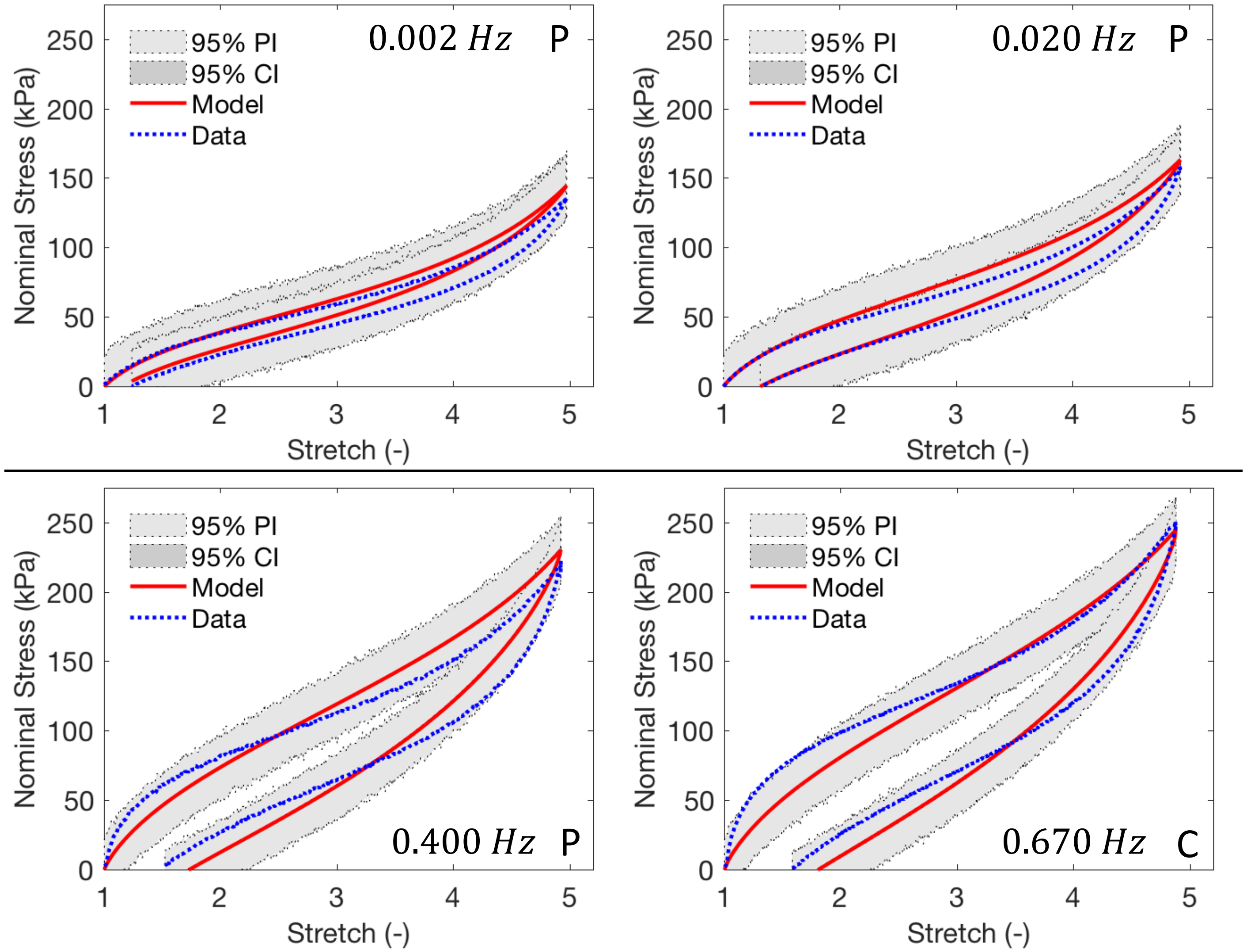
Uncertainty Propagation: VHB 4949

- C – Calibrated
- P – Predicted

- Linear viscoelasticity
- Fractional order rate relation

$$Q_{iK} = \eta D_t^{\alpha} F_{iK}$$

Calibrated Rate (1/s)	$\bar{\eta}$ (kPa·s)	$\bar{\alpha}$ (-)
0.002	996	0.81
0.020	63.4	0.45
0.400	41.8	0.33
0.670	53.4	0.30



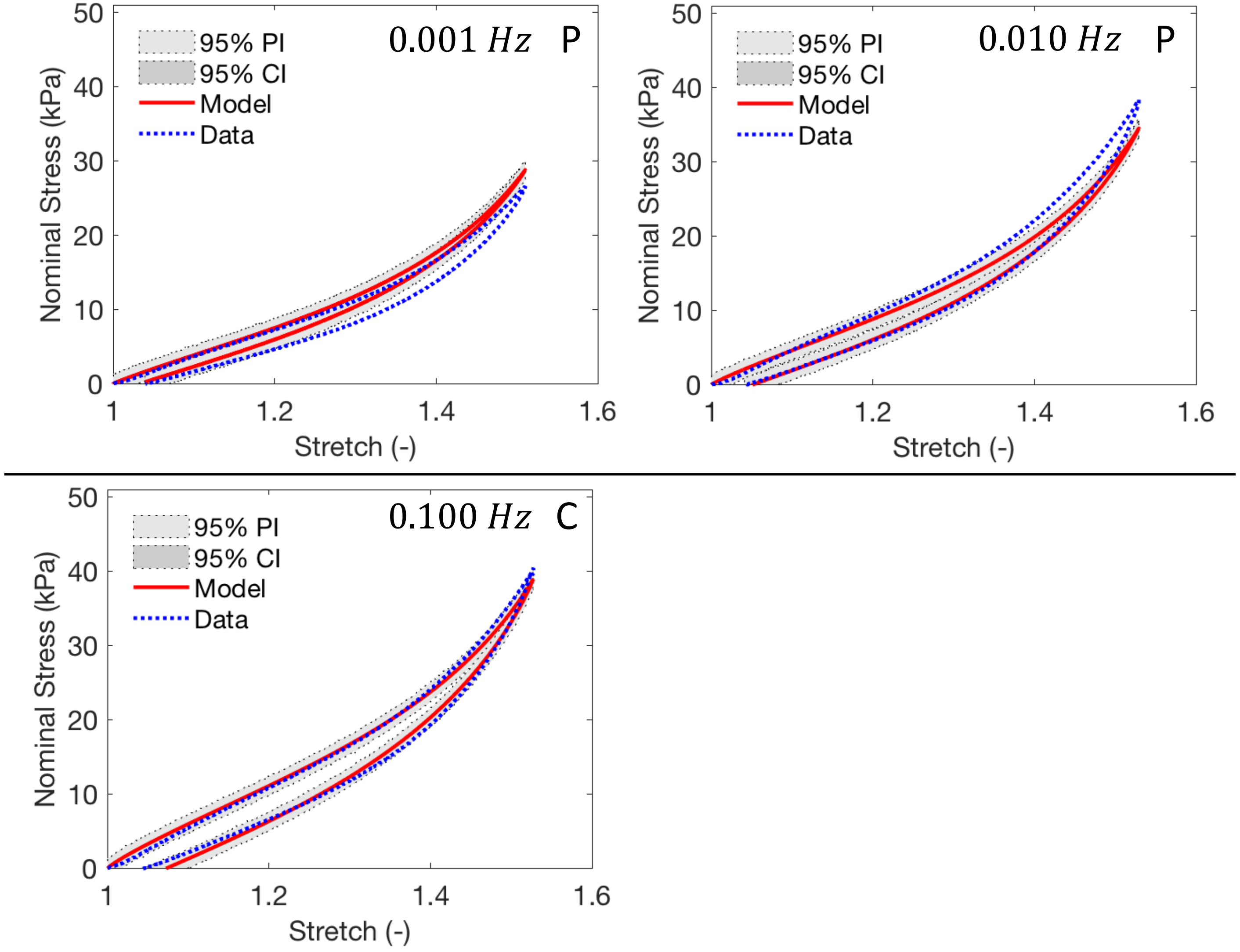
Uncertainty Propagation: Auxetic Foam

- C – Calibrated
- P – Predicted

- Linear viscoelasticity
- Fractional order rate relation

$$Q_{iK} = {\color{red}\eta} D_t^{\color{red}\alpha} F_{iK}$$

Calibrated Rate (1/s)	$\bar{\eta}$ (kPa·s)	$\bar{\alpha}$ (-)
0.001	266	0.69
0.010	44.1	0.27
0.100	29.5	0.24



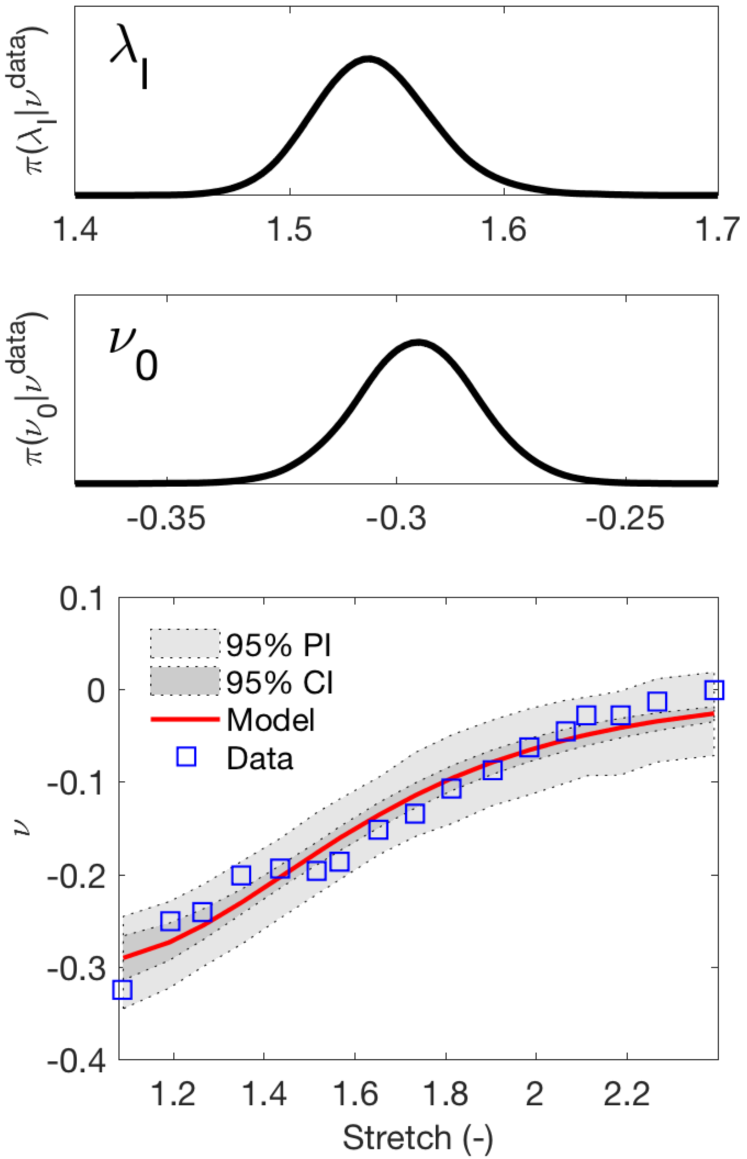
Modeling Negative Poisson Ratio

- Experiment – digital processing
- Blatz-Ko

$$\nu = \nu_0 (1 + \pi^2 \nu_0^2 (\lambda - 1)^2)^{-q}$$

$$q = \frac{1}{2} + \frac{1}{2\pi^2 (\lambda_I - 1)^2 \nu_0^2}$$

- ν_0 - infinitesimal Poisson ratio
- λ_I - transition from expansion to contraction



Conclusions & Future Work: Fractional Viscoelasticity

- Material dependent fractional order α
- Soft elastomers: VHB 4910 & 4949
  - Predicts behavior over broad range of operating regimes
- Auxetic foam:
  - Reasonable prediction across range of rates tested
  - Investigate impact of auxetic nature on elastic response

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