Viscoelastic Modeling Guided by Uncertainty Quantification

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Outline

- Motivation
- Experimental setup & results
- Model theory
- Model development
  - Linear vs. nonlinear viscoelasticity
  - Bayesian uncertainty analysis
- Conclusions

Motivation

- **Application of VHB 4910 (made by 3M)**
  - Construction, robotics, flow control
- **Damping:**
  - Shock absorption via materials
  - Characterization required to develop dynamic tunability

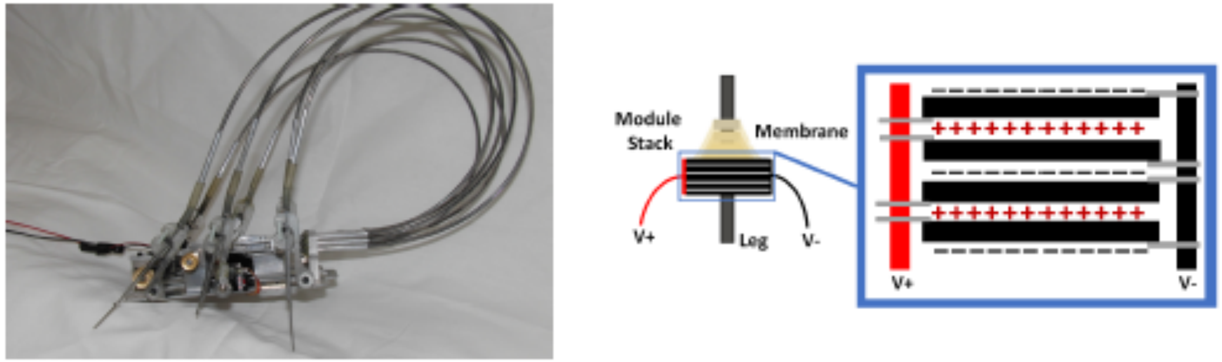
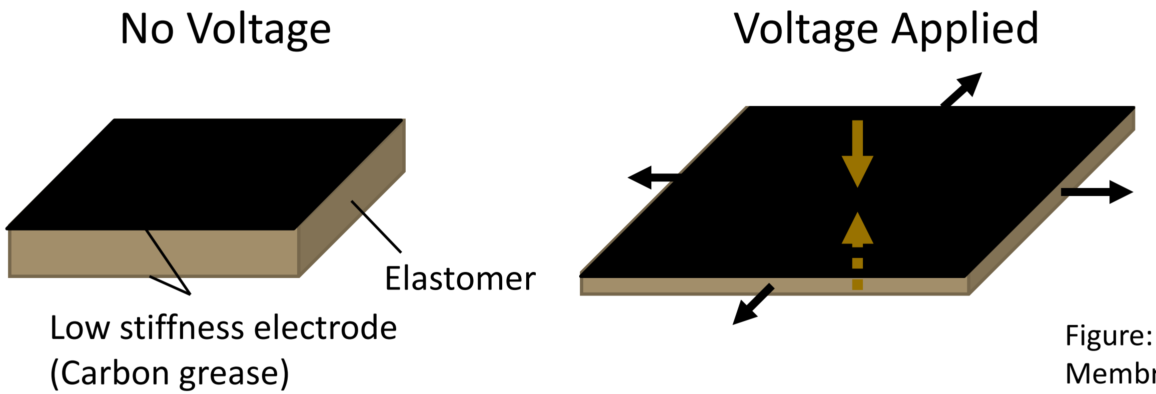


Figure: iSprawl robotic platform (from Newton, J., *"Design and Characterization of a Dielectric Elastomer Based Variable Stiffness Mechanism for Implementation onto a Dynamic Running Robot,"* (2014), Figure 2.11 and Figure 4.5.)



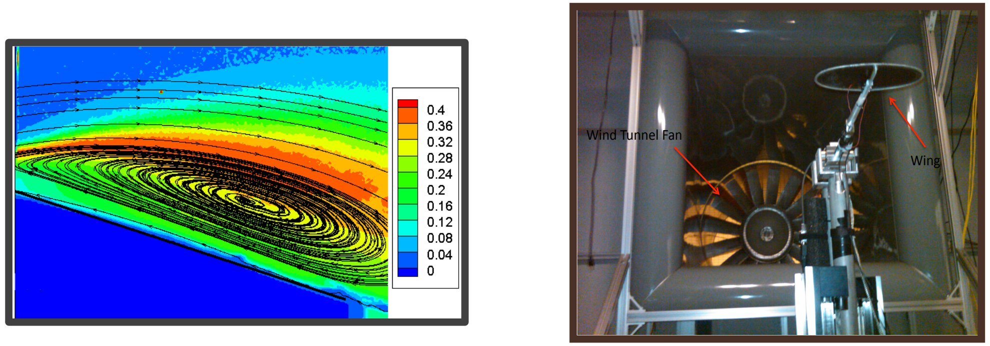


Figure: Hays, et al. "Aerodynamic Control of Micro Air Vehicle Wings Using Electroactive Membranes," J. Mater. Syst. Struct., v. 24(7), pp. 862-878, 2013.

Motivation

Two important questions:

- What is the most appropriate hyperelastic model?
  - Ogden, Mooney-Rivlen, Langevin, Nonaffine, etc.
- How does hyperelasticity couple with viscoelasticity?
  - Thermodynamic perspective
  - Linear vs. nonlinear viscoelasticity

Experimental Setup

- **Tests:**
  - Performed at different stretch rates
  - Specimens taken through a stretch/relaxation cycling to remove viscoplastic effects (explained on next slide).
- **Stretch rate:** $\dot{\lambda} = \frac{\dot{x}}{L_0}$
  - Where
    $\dot{\lambda}$ is the stretch rate (Hz)
    $\dot{x}$ is the speed of the moving clamp head (mm/s)
    L_0 is the initial length of the VHB specimen (mm)

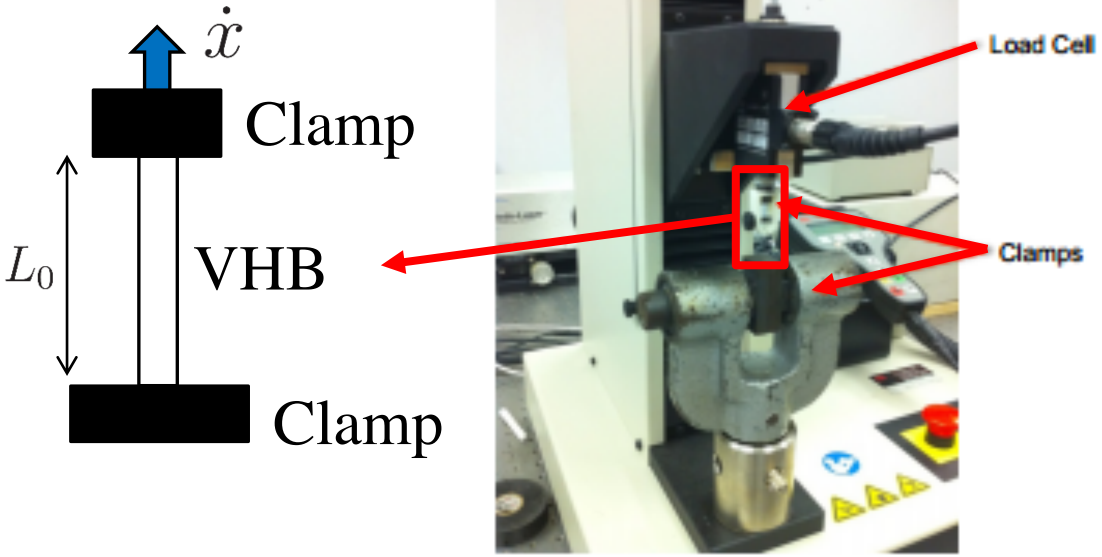
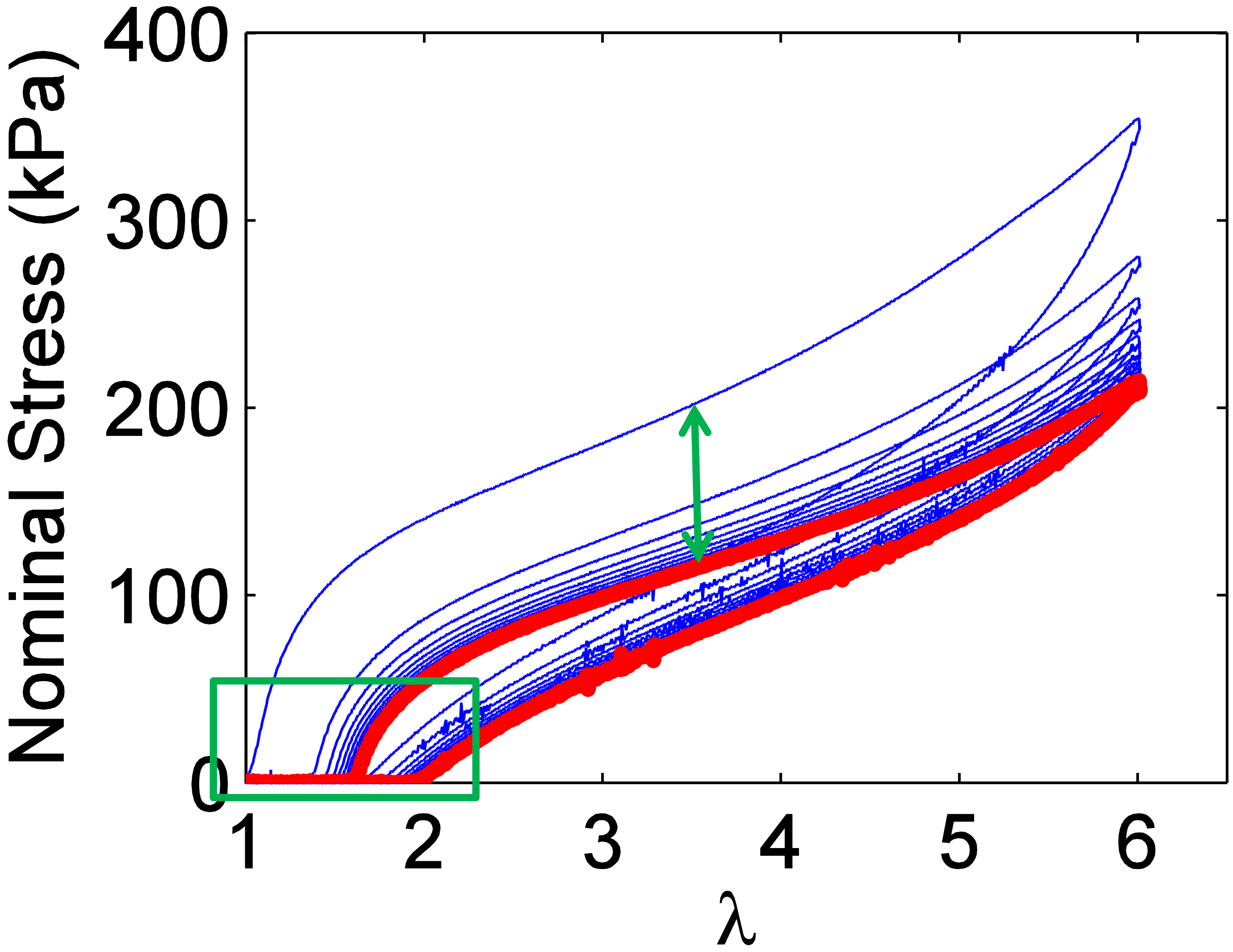
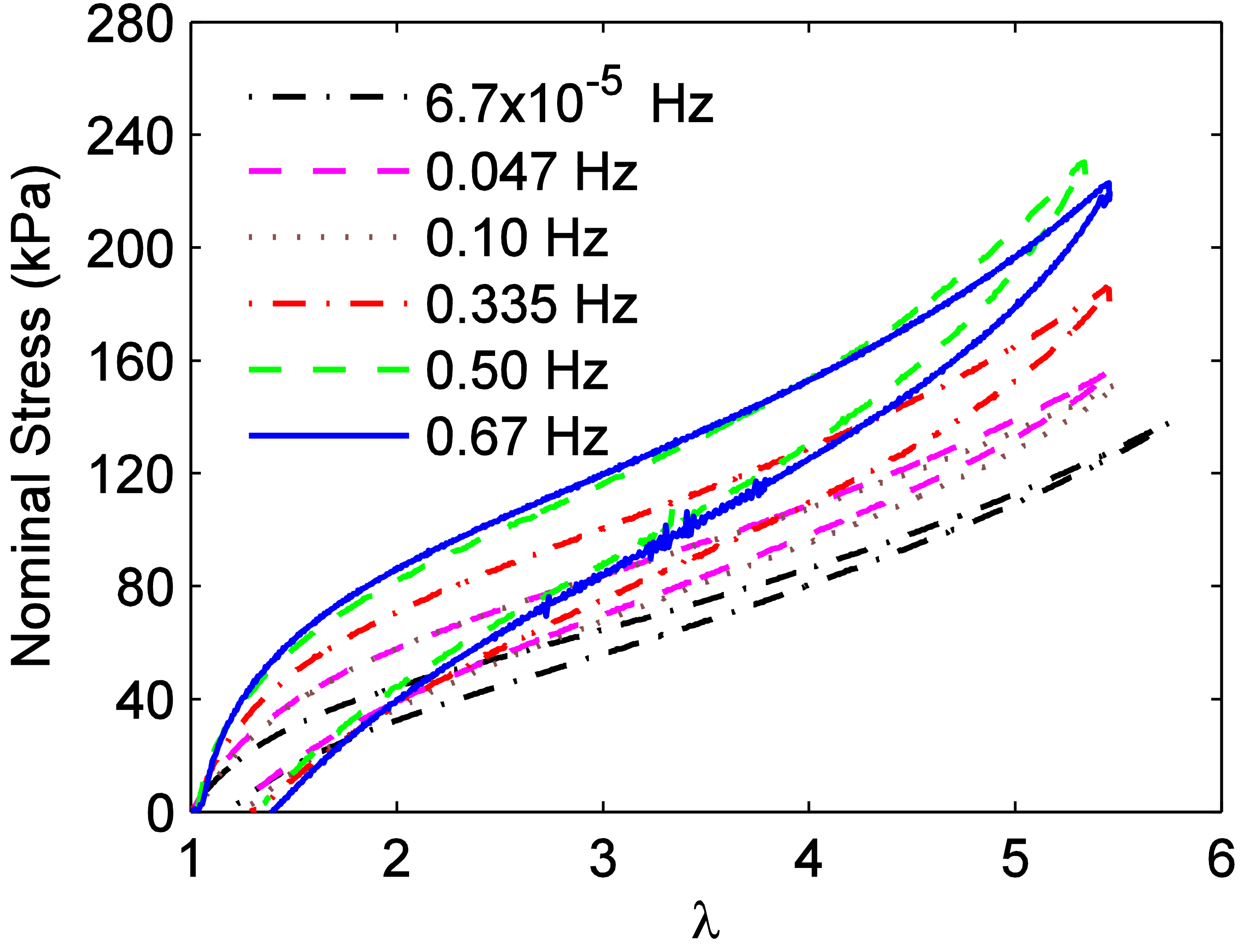


Figure: MTS tensile testing of VHB 4910 (from Morton, J., "*Control of the Stiffness of Robotic Appendages Using Dielectric Elastomers*," Masters Thesis, 2012)

Cyclic Loading of VHB 4910



12th Cycle for All Stretch Rates



Model Theory

- **1st Law of Thermodynamics**

$$\rho^0 \dot{\hat{\psi}} = s_{iK}\dot{F}_{iK} + \rho^0 r - Q_{I,I} - \rho^0 \Theta \dot{S} - \rho^0 S \dot{\Theta}$$

- **2nd Law of Thermodynamics**

$$\rho^0 \dot{S} \geq \frac{\rho^0 r}{\Theta} - \frac{1}{\rho^0}\left(\frac{Q_I}{\Theta}\right)_{,I} \qquad -\frac{\partial \hat{\psi}}{\partial \Gamma^\alpha_{iK}}\dot{\Gamma}^\alpha_{iK} - \frac{Q_I \Theta_{,I}}{\Theta^2} \geq 0$$

Entropy generation

$$-\frac{\partial \hat{\psi}}{\partial \Gamma^\alpha_{iK}}\dot{\Gamma}^\alpha_{iK} = Q^\alpha_{iK}\dot{\Gamma}^\alpha_{iK} = F(\dot{\Gamma}^\alpha_{iK}, F_{iK}) \geq 0$$

Viscoelastic stress

Model Theory

Hyperelastic: Ogden

$$\psi_\infty^O = \sum_{p=1}^{3} \frac{\mu_p}{\alpha_p} \left(I_1^{\alpha_p} - 3 \right)$$

Hyperelastic: Nonaffine

$$\psi_\infty^N = \frac{1}{6} G_c I_1 - G_c \lambda_{max}^2 ln(3\lambda_{max}^2 - I_1) + G_e \sum_j (\lambda_j + \frac{1}{\lambda_j})$$

Davidson & Goulbourne, J. Mech. Phys. Solids, (2013), v. 61(8), pp. 1784-1797.

Viscoelastic: Linear (Spring-Dashpot)

$$\Upsilon_L = \sum_\alpha \left[\frac{1}{2} \gamma^\alpha \left(F_{iK} - \Gamma_{iK}^\alpha \right) \left(F_{iK} - \Gamma_{iK}^\alpha \right) \right]$$

Viscoelastic: Nonlinear

$$\Upsilon_{NL} = \sum_\alpha \left[\frac{1}{2} \gamma_\alpha \Gamma_{iK}^\alpha \Gamma_{iK}^\alpha - \beta_\infty^\alpha \frac{\partial \psi_\infty}{\partial F_{iK}} \Gamma_{iK}^\alpha + \beta_\infty^\alpha \psi_\infty \right]$$

Holzapfel & Simo, Int. J. Solid Struct., (1996), v. 33(20-22), pp. 3019-3034.

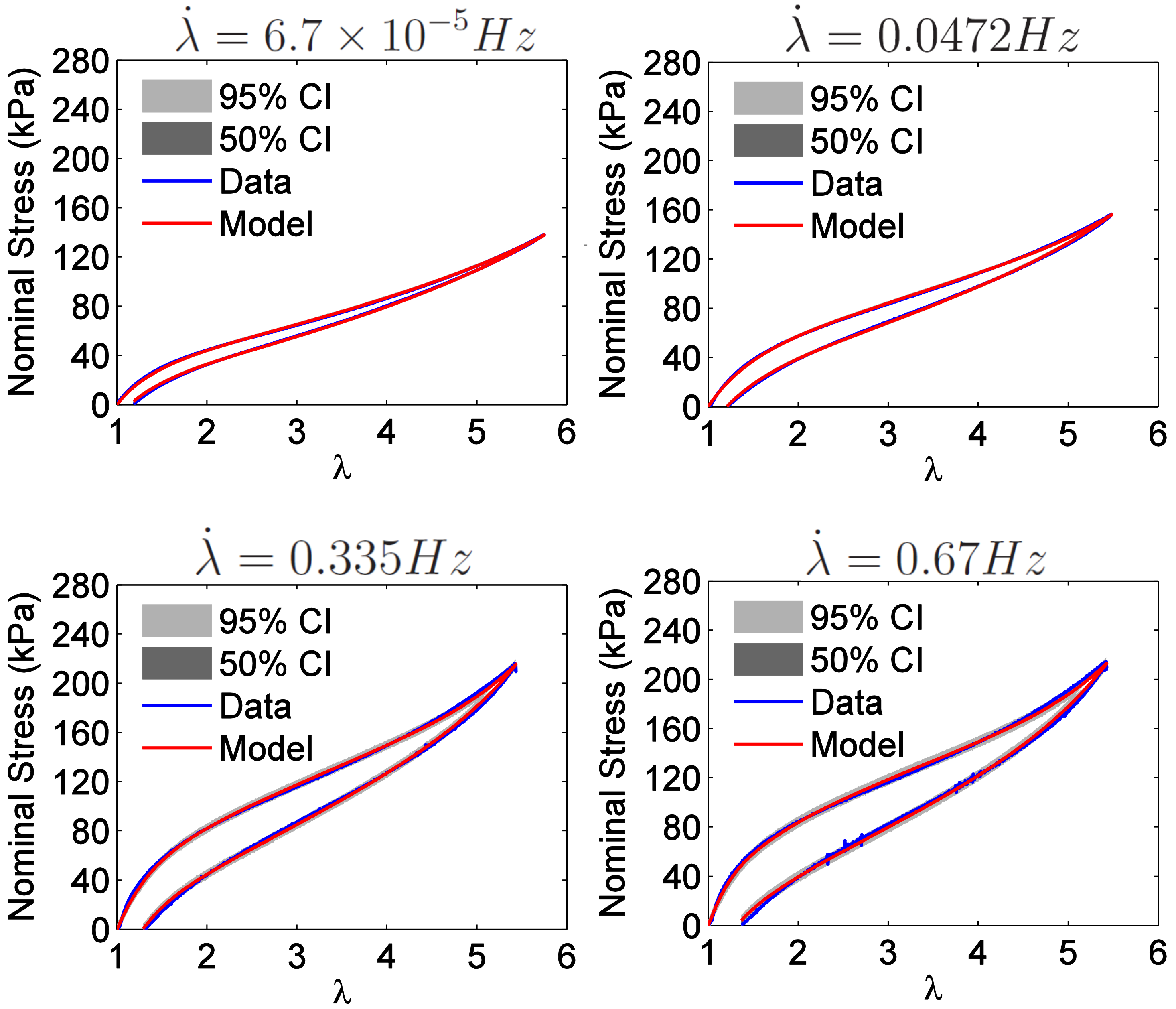
Bayesian Uncertainty Analysis

- Optimization performed using Markov Chain Monte Carlo algorithm (MCMC).
  - Random sampling
  - Global optimization
  - Parameter Relationships

- Bayes' Relation
  - $\pi(\theta|y) = \frac{p(y|\theta)\pi_0(\theta)}{\int_{\mathbb{R}^p} p(y|\theta)\pi_0(\theta)d\theta}$

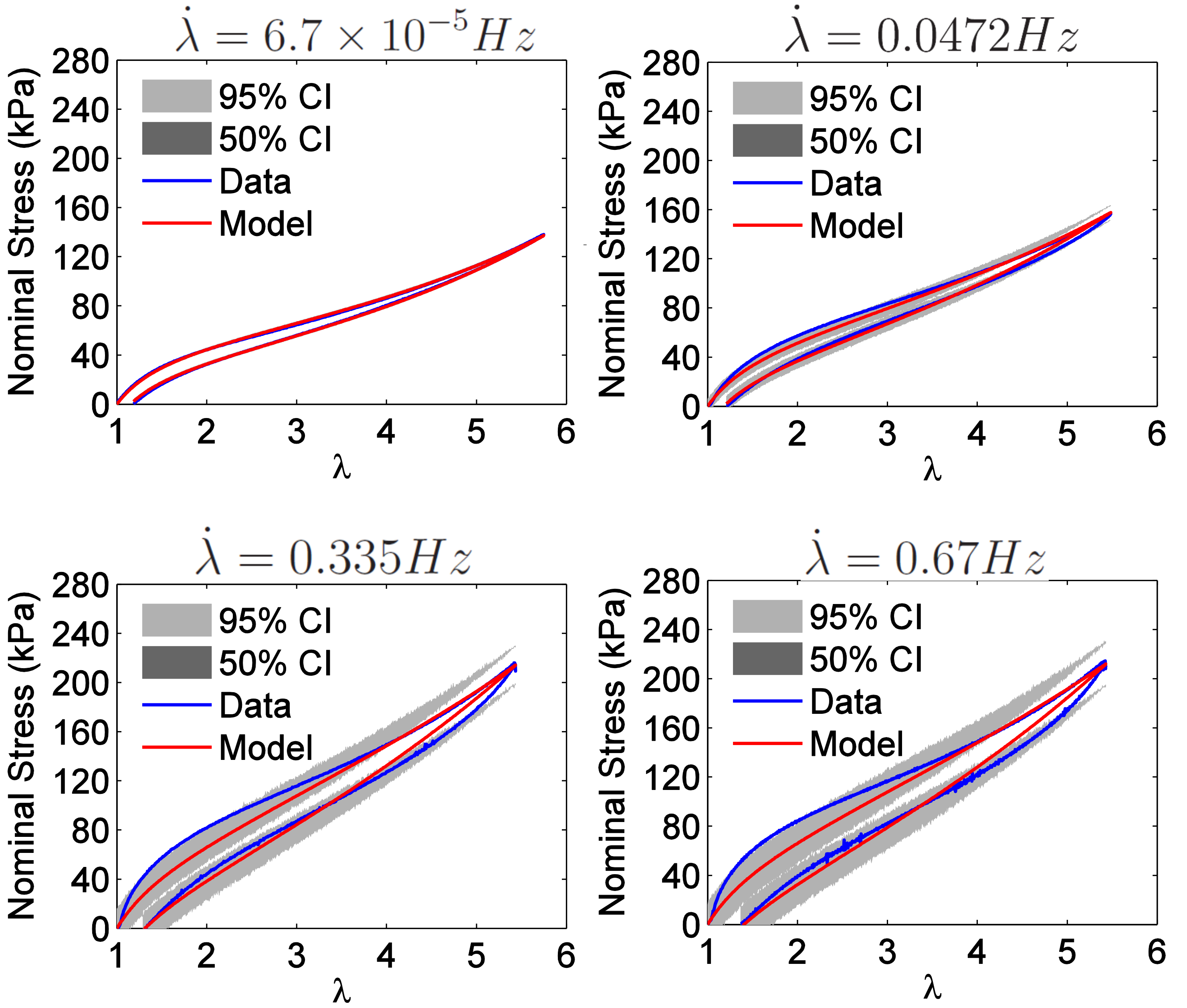
Results

- **Hyperelastic:**
  - Ogden
  - Open
- **Viscoelastic:**
  - Linear
- Results shown for 4 stretch rates



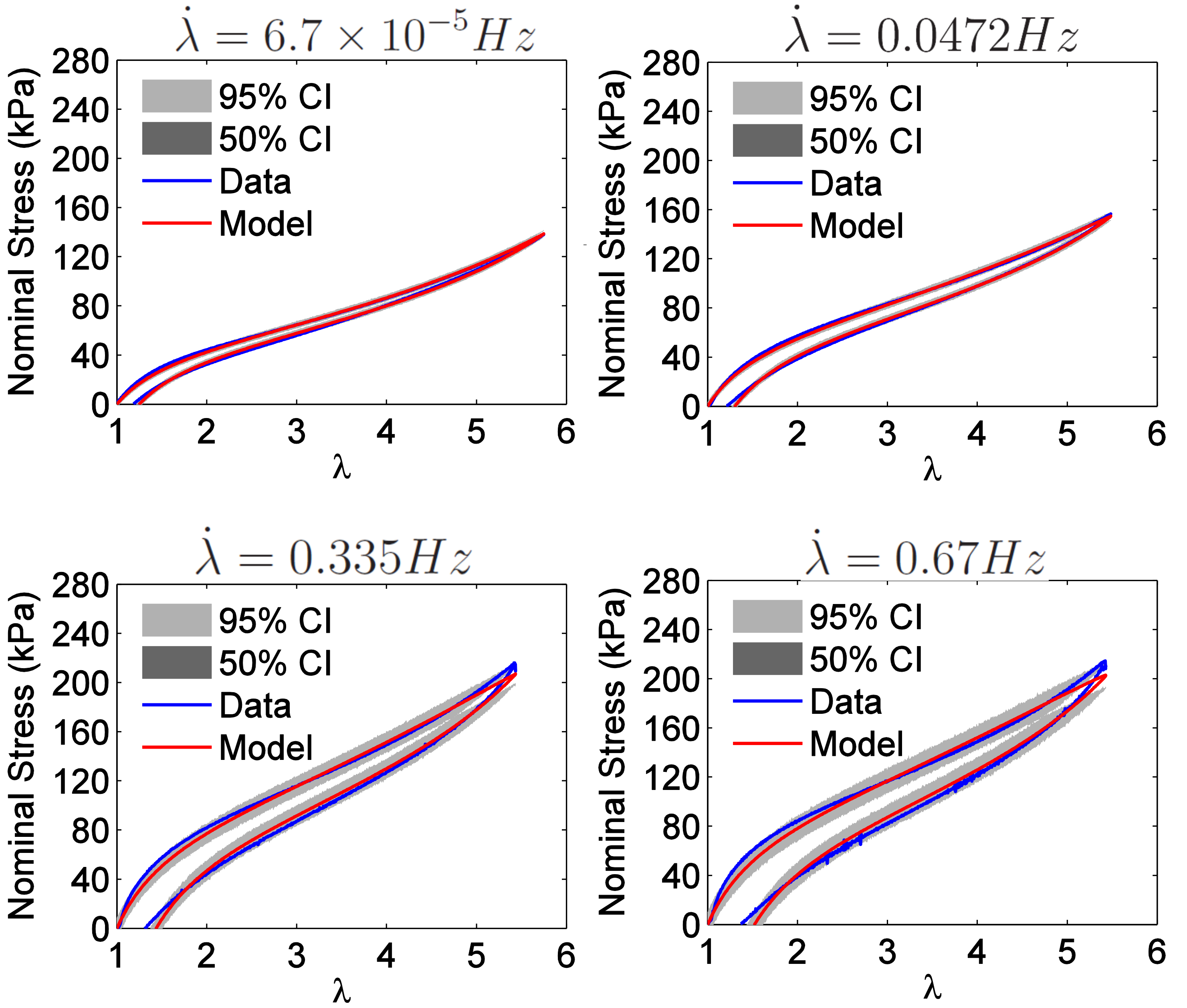
Results

- **Hyperelastic:**
  - Ogden
  - Fixed
- **Viscoelastic:**
  - Linear
- Results shown for 4 stretch rates



Results

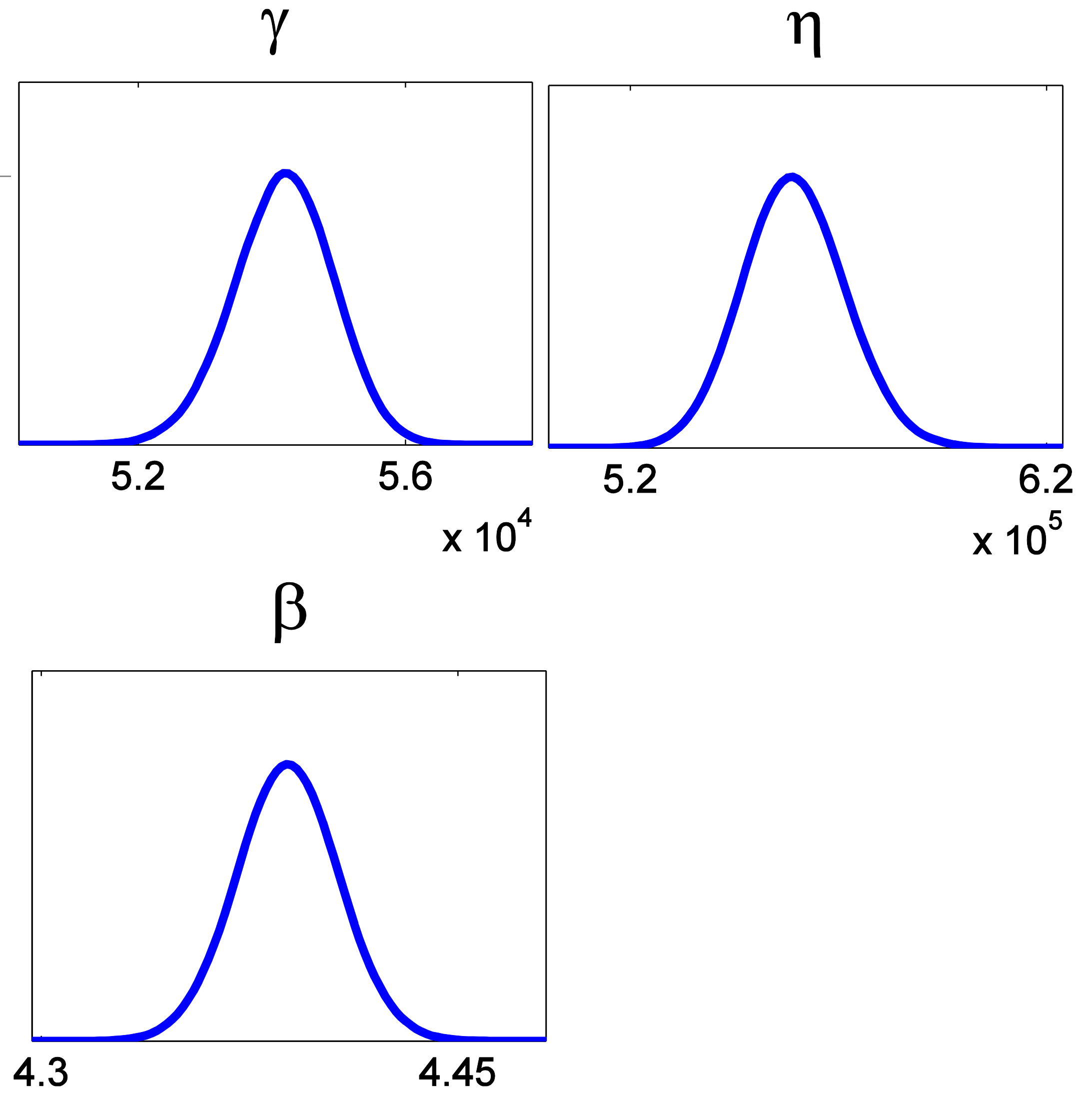
- **Hyperelastic:**
  - Nonaffine
  - Fixed
- **Viscoelastic:**
  - Nonlinear
- Results shown for 4 stretch rates



Hyperelastic	Ogden	Nonaffine		Ogden	Nonaffine	
Viscoelastic	Linear	Linear	Nonlinear	Linear	Linear	Nonlinear
Optimization	Open			Fixed		
Stretch Rate	All error is calculated in kPa^2.					
6.7×10^{-5} Hz	0.50	0.87	1.86	0.50	0.87	1.86
0.0472 Hz	0.60	0.68	2.69	9.34	11.0	2.71
0.10 Hz	0.67	0.81	2.96	14.6	16.1	3.03
0.335 Hz	3.50	4.05	14.4	62.9	67.8	17.2
0.50 Hz	4.40	5.70	14.6	54.4	59.2	33.0
0.67 Hz	4.80	6.42	22.5	82.1	86.6	25.7

Parameter Distributions

- Distribution developed from sampled parameters.
- Example distribution results:
  - Nonaffine, nonlinear
  - Fixed hyperelastic parameters
  - Fastest stretch rate (0.67 Hz)

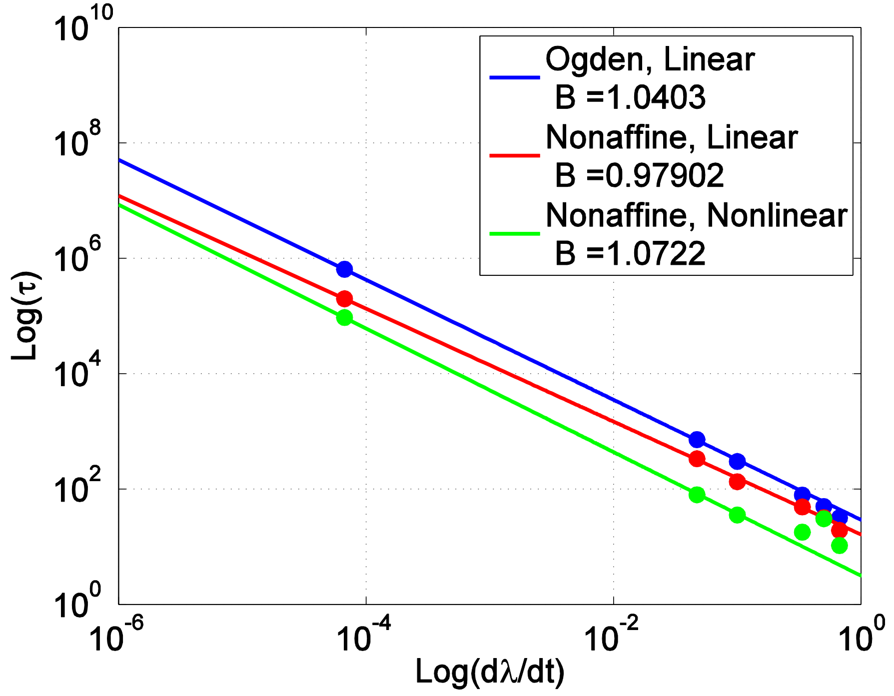


Conclusions and Future Work

- **Conclusions:**
  - Quantified <u>stretch rate independent</u> hyperelastic parameters
  - Coupled hyperelasticity with linear and nonlinear viscoelasticity
  - Nonlinear viscoelastic model ~3X more accurate for VHB4910

- **Future:**
  - Model validation over stretch rates not tested
  - Nonlinear finite element implementation for complex loading
  - Incorporate effects of viscoplasticity
  - Multiscale statistical analysis of polymer networks

Power law relation on time constant



Acknowledgements

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