Uncertainty Analysis of Dielectric Elastomer Membranes Under Multi-Axial Loading

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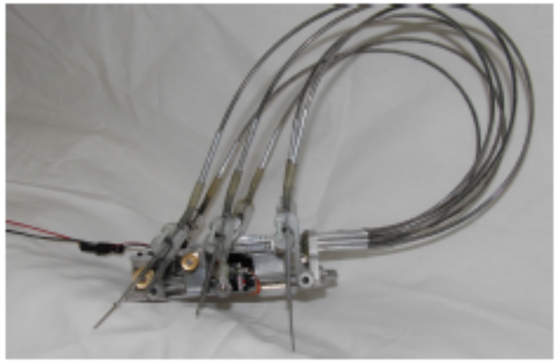
Outline

- Motivation
- Experiment:
  - Setup
  - Results
- Model:
  - Development
  - Calibration
- Conclusions

Motivation

- Applications of VHB 4910 (made by 3M)
  - Robotics & Flow Control[1]
- Multi-physics
  - Explore calibration of shared characteristics



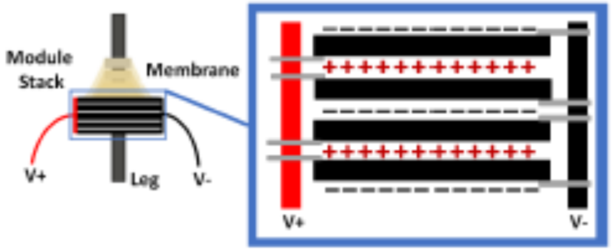
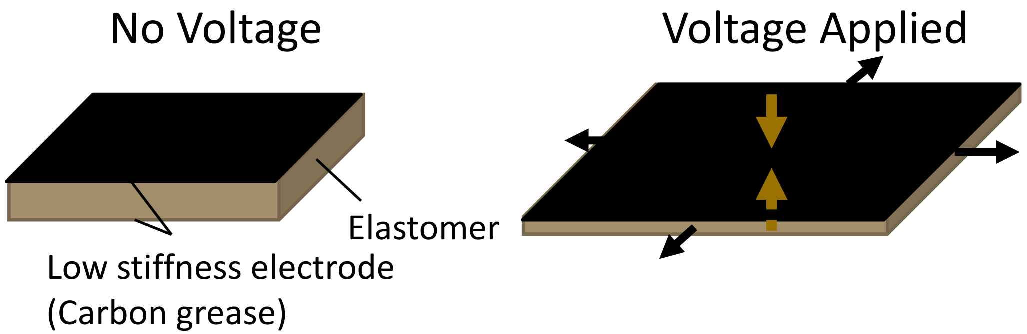
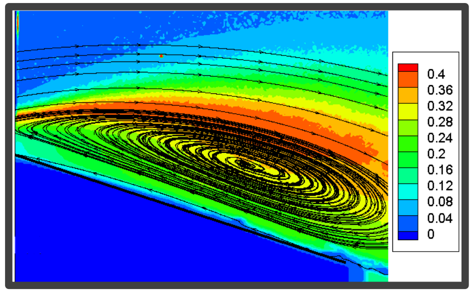


Figure: iSprawl robotic platform (from Newton, J., "*Design and Characterization of a Dielectric Elastomer Based Variable Stiffness Mechanism for Implementation onto a Dynamic Running Robot,*" (2014), Figure 2.11 and Figure 4.5.)





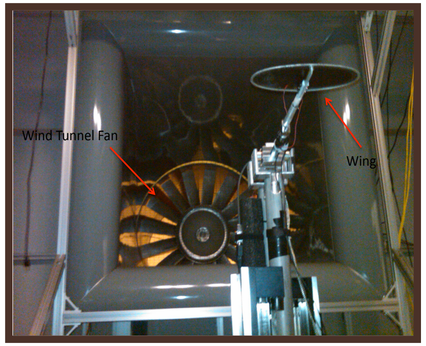
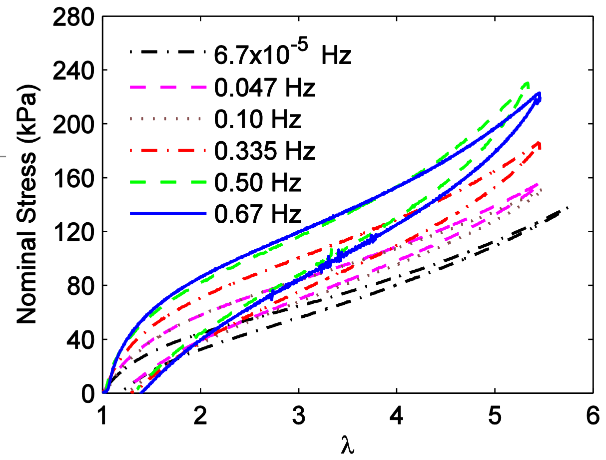


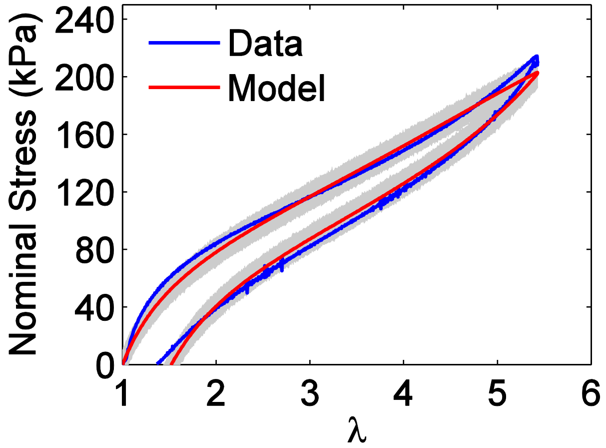
Figure: Hays, et al. "Aerodynamic Control of Micro Air Vehicle Wings Using Electroactive Membranes," J. Mater. Syst. Struct., v. 24(7), pp. 862-878, 2013.

1. O'Halloran, Ailish, Fergal O'Malley, and Peter McHugh. "A review on dielectric elastomer actuators, technology, applications, and challenges." Journal of Applied Physics 104.7 (2008): 071101.

Motivation

- Project
  - Uncertainty quantification has been applied to a wide variety of disciplines:
    - Atomistic Potentials[1], Computational Fluid Dynamics[2], Weather Prediction[3]
  - Less work done in quantifying uncertainty of material models.
  - Previous work has been done to characterize the hysteresis in VHB 4910 through of series of uni-axial experiments[4].
  - Quantifying uncertainty under multi-axial electromechanical loading.
- Challenges
  - Model calibration of two different types of data
  - Appropriate model





1. Frederiksen, Søren L., et al. "Bayesian ensemble approach to error estimation of interatomic potentials." Physical review letters 93.16 (2004): 165501.
2. Croicu, Ana-Maria, et al. "Robust Airfoil Optimization Using Maximum Expected Value and Expected Maximum Value Approaches." AIAA journal 50.9 (2012): 1905-1919.
3. Wilks, Daniel S. Statistical methods in the atmospheric sciences. Vol. 100. Academic press, 2011.
4. Miles, Paul, et al. "Bayesian uncertainty analysis of finite deformation viscoelasticity." Mechanics of Materials 91 (2015): 35-49.

Experimental Setup

- Mechanical: Transverse loading
  - Load monitored for prescribed displacement
  - Triangular load/unload cycle
  - Test cases: 0-6 kV

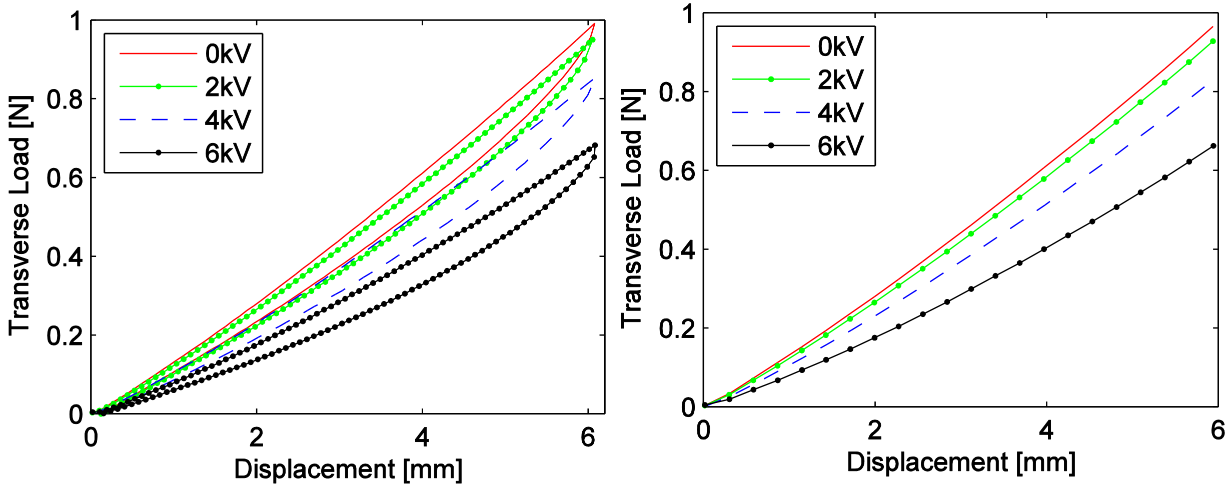


Figure: Transverse load data. (a) Complete load/unload cycle and (b) load cycle used for model calibration.

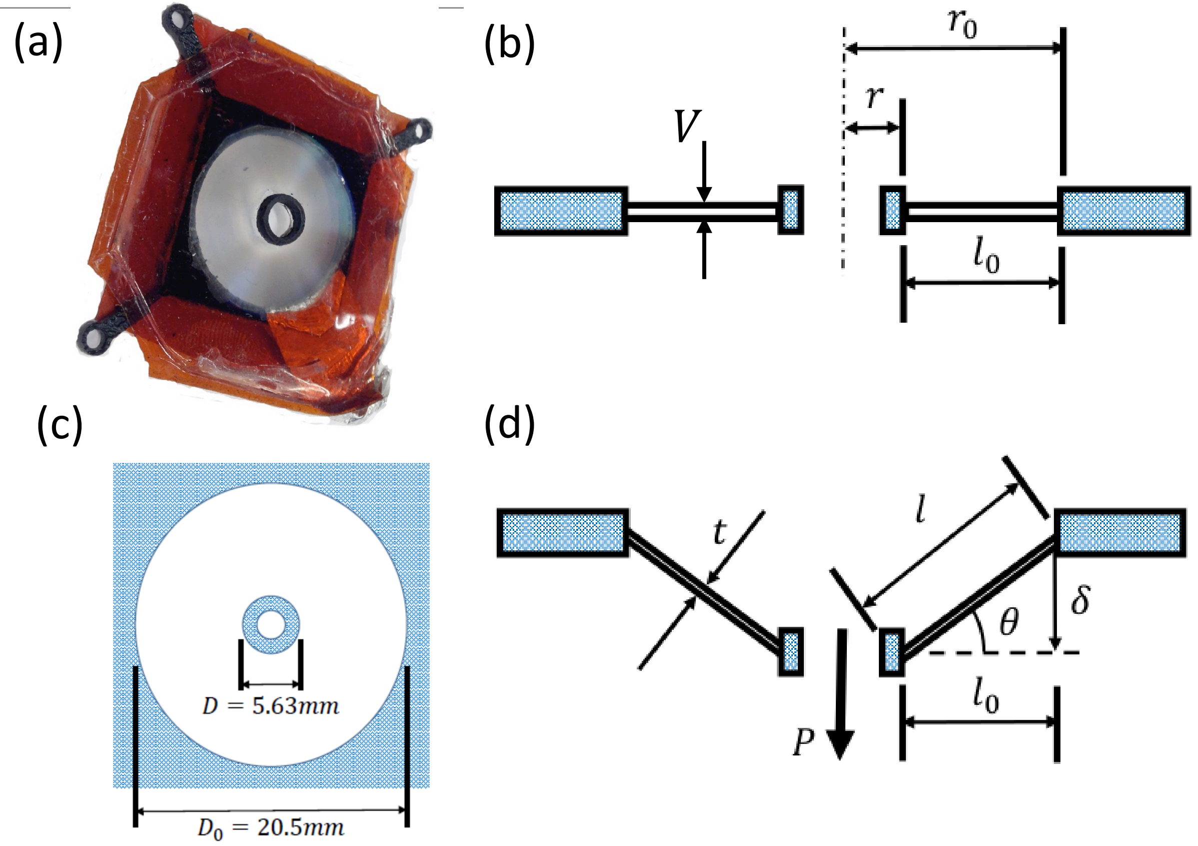


Figure: Schematic of problem geometry, transverse loading and material deformation. (a) VHB Specimen (b) Non-deformed configuration (c) Overhead view (d) Deformed configuration.

Experimental Setup

- Electrical: Material Capacitance
  - Sawyer-Tower circuit
  - Connected in series with a known capacitor
  - Performed in non-deformed configuration

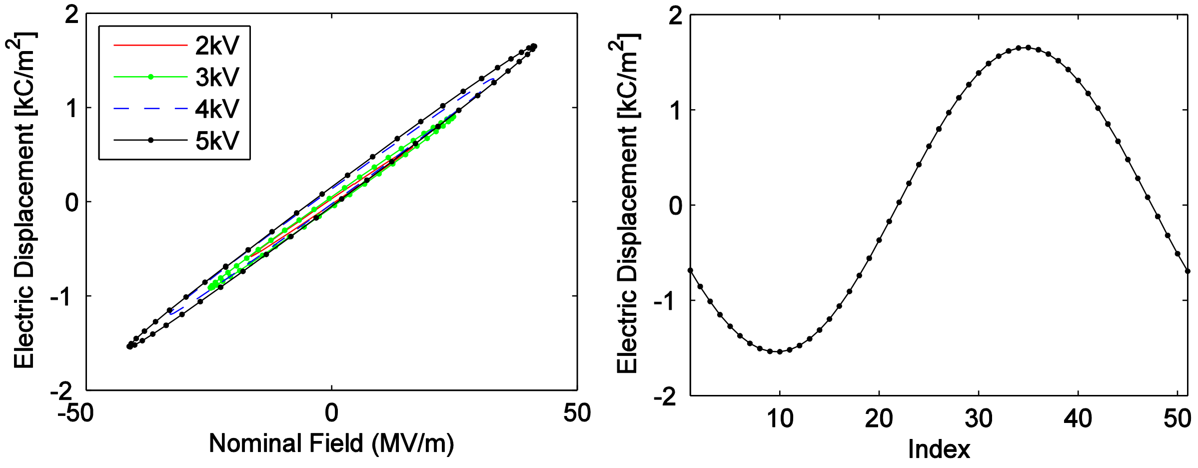
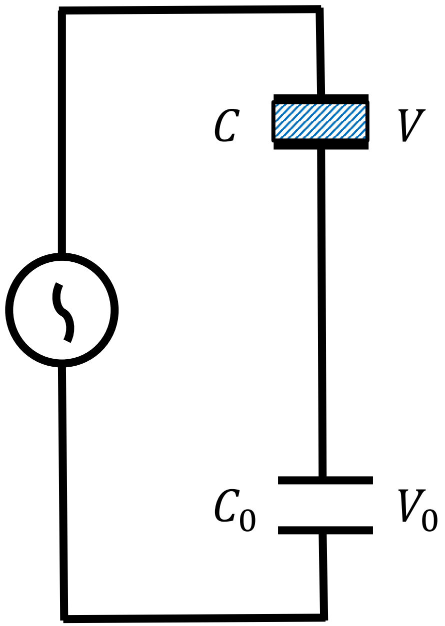
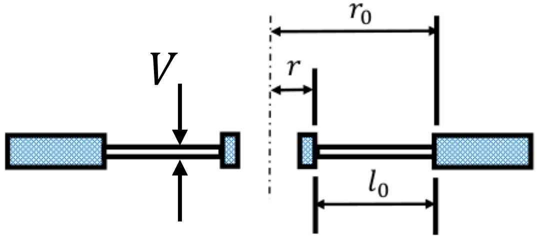


Figure: Data collected from Sawyer-Tower circuit. (Left) Electric displacement plotted as a function of the nominal field and (Right) electric displacement as a function of index from a single loop.

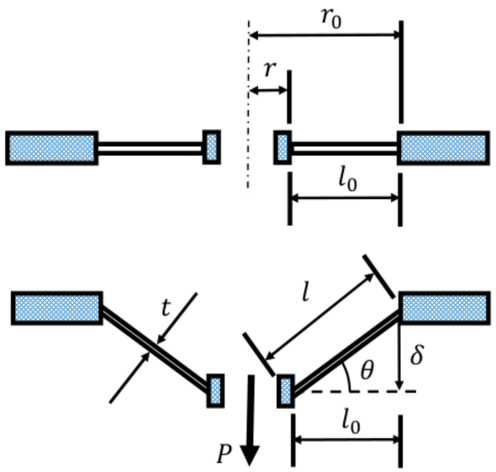




A sinusoidal voltage is applied at a frequency of 1 Hz for 5 cycles. The voltage is sent through a linear TREK amplifier and measurements across the capacitor are performed with a Keithley 6517A electrometer.

Figure: Sawyer-Tower circuit with VHB in series with 153 μF capacitor (C_0). VHB specimen measured from non-deformed configuration.

Modeling – Transverse Load



- Transverse load[1]

$$F = 2\pi \sin(\theta)\, r t \sigma_l$$

- Cauchy stress in radial direction. Application of electric field in transverse direction decreases the Cauchy stress[2].

$$\sigma_l = \sigma_l^H - \kappa_r \epsilon_0 E_t^2$$

Relative permittivity, κ_r, is assumed independent of deformation

- Nonaffine hyperelastic stress assuming incompressibility[3]
  - $\lambda_{i,tot} = \lambda_{i,pre}\lambda_i$ and $\sum_i \lambda_{i,tot} = 1$ where $i = l, c, t$ (radial, circumferential, thickness)

$$\sigma_l^H = \frac{G_c}{3}\left(\lambda_{l,tot}^2 - \frac{1}{\lambda_{c,pre}^2 \lambda_{l,tot}^2}\right)\left(\frac{9\lambda_{max}^2 - I_1}{3\lambda_{max}^2 - I_1}\right) + G_e\left(\lambda_{l,tot}(1 + \lambda_{c,pre}) - \frac{1 + \lambda_{c,pre}}{\lambda_{c,pre}\lambda_{l,tot}}\right)$$

1. Rizzello, Gianluca, et al. "Dynamic Electromechanical Modeling of a Spring-Biased Dielectric Electroactive Polymer Actuator System." ASME 2014 Conference on Smart Materials, Adaptive Structures and Intelligent Systems. American Society of Mechanical Engineers, 2014.
2. Zhao, Xuanhe, Wei Hong, and Zhigang Suo. "Electromechanical hysteresis and coexistent states in dielectric elastomers." Physical review B 76.13 (2007): 134113.
3. Davidson, Jacob D., and N. C. Goulbourne. "A nonaffine network model for elastomers undergoing finite deformations." Journal of the Mechanics and Physics of Solids 61.8 (2013): 1784-1797.

Modeling – Electric Displacement

- Polarization model[1]

$$\ddot{P}_t + \gamma \dot{P}_t + \frac{K}{m} P_t = \frac{Ne^2}{m} E_t$$

- True electric displacement is related to the electric field by

$$D_t = \epsilon_0 E_t + P_t$$

Note the nominal and true electric field should be the same under the assumption that the membrane does not buckle.

- Ignoring 2nd order rate effects yields the rate-dependent dielectric constitutive model

$$\tau \dot{D}_t + D_t = \tau \epsilon_0 \dot{E}_t + \kappa_r \epsilon_0 E_t$$

where $\kappa_r \epsilon_0 = 1 + \frac{Ne^2}{K\epsilon_0}$ and $\tau = \frac{\gamma m}{K}$.

1. Fowles, Grant R. Introduction to modern optics. Courier Corporation, 2012.

Bayesian Uncertainty Analysis

- Calibration: Markov Chain Monte Carlo (MCMC)
  - Random sampling
- Bayes' Relation
  - $\pi(\theta|M^{data}) = \frac{p(M|\theta)\pi_0(\theta)}{\int_{\mathbb{R}^p} p(M|\theta)\pi_0(\theta)d\theta}$

$\pi(\theta|M^{data})$ - posterior density
$\pi_0(\theta)$ - prior density (*a priori* knowledge)
$p(M|\theta)$ - likelihood of model given parameters

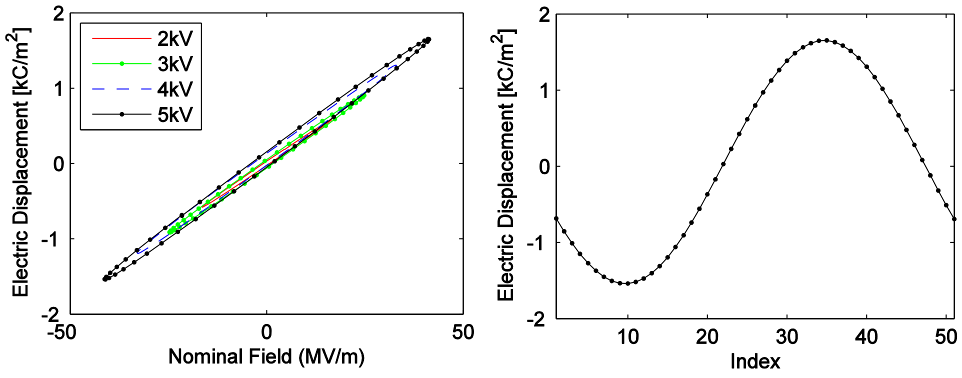
- Likelihood: $p(M|\theta) = e^{-\sum_{i=1}^{n}[M^{data}(i)-M(i;\theta)]^2/(2\sigma^2)}$
  - Assume observation errors are independent and identically distributed (iid) and $\varepsilon_i \sim N(0,\sigma^2)$.

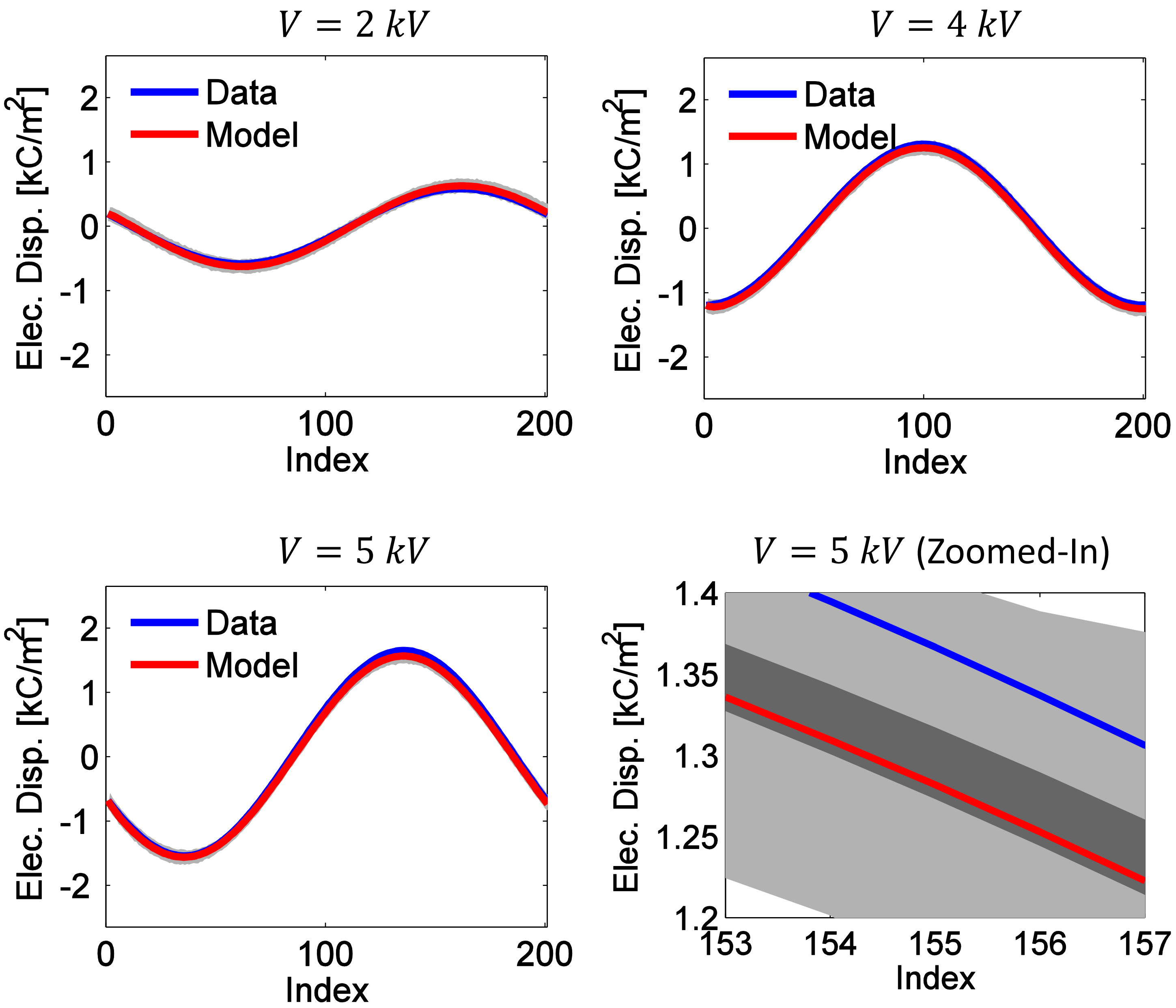
- Decoupled Problem:

1) Use electric displacement data: $\theta = [\kappa_r, \tau]$
2) Use transverse load data with no applied voltage: $\theta = [G_c, G_e, \lambda_{max}]$
3) Use all data from both experiments – reformulate model to energy: $\theta = [G_c, G_e, \lambda_{max}, \kappa_r, \tau]$

Results (1)

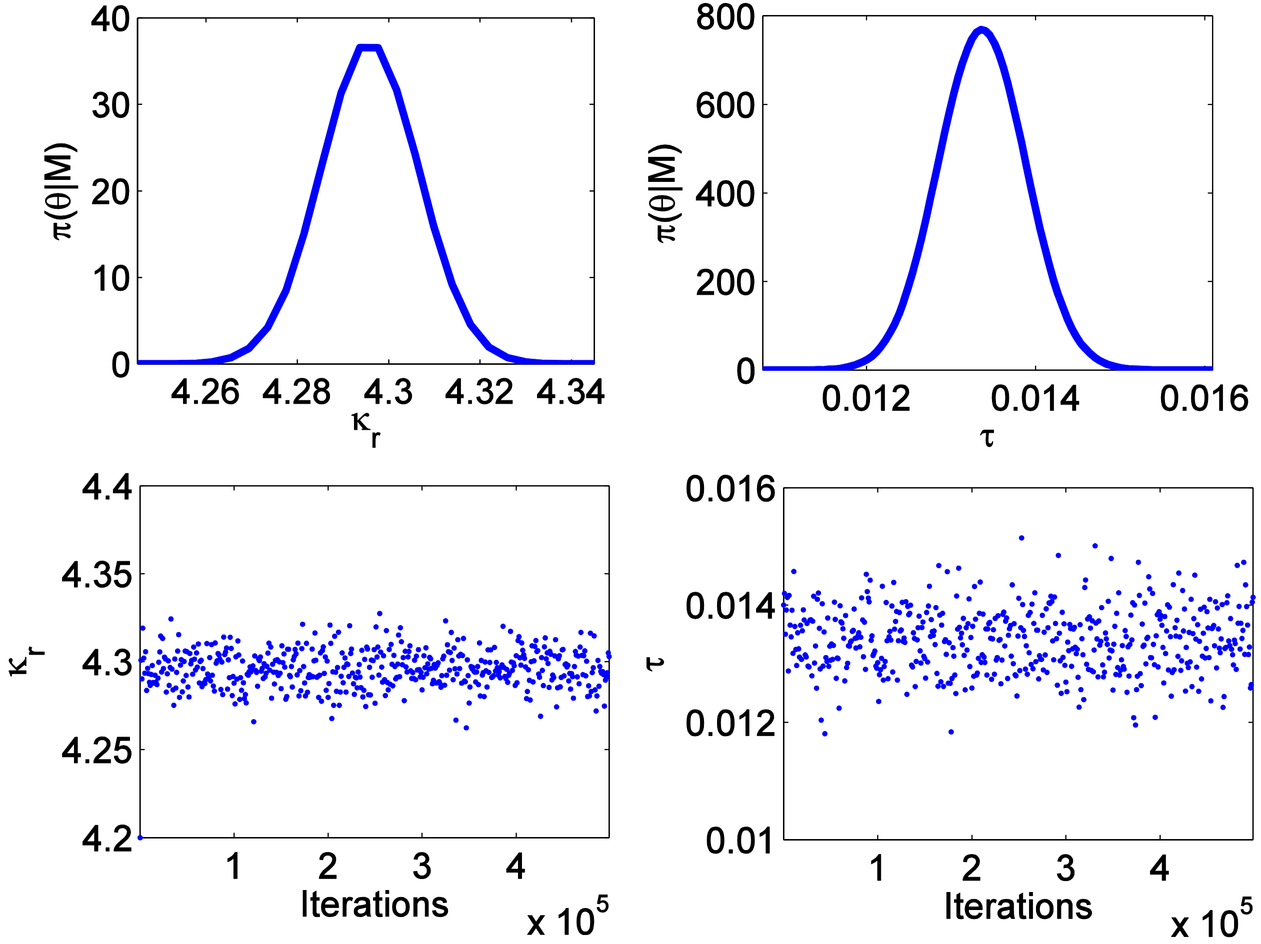
- Electric Displacement
  - Magnitude of electric displacement increases with applied voltage
  - Prediction intervals shown in light grey and credible intervals shown in dark grey





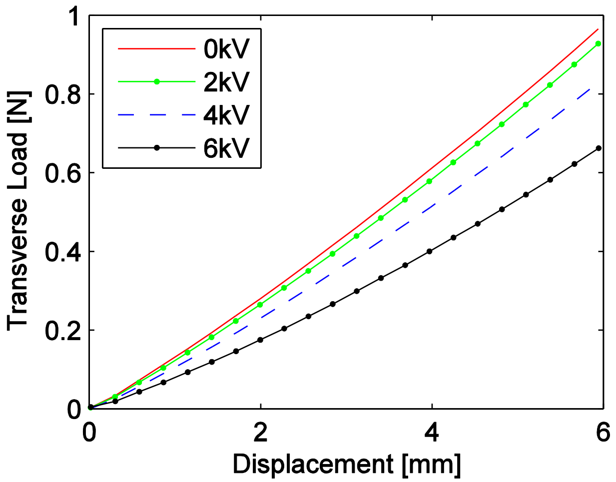
Parameter Distributions & Chains (1)

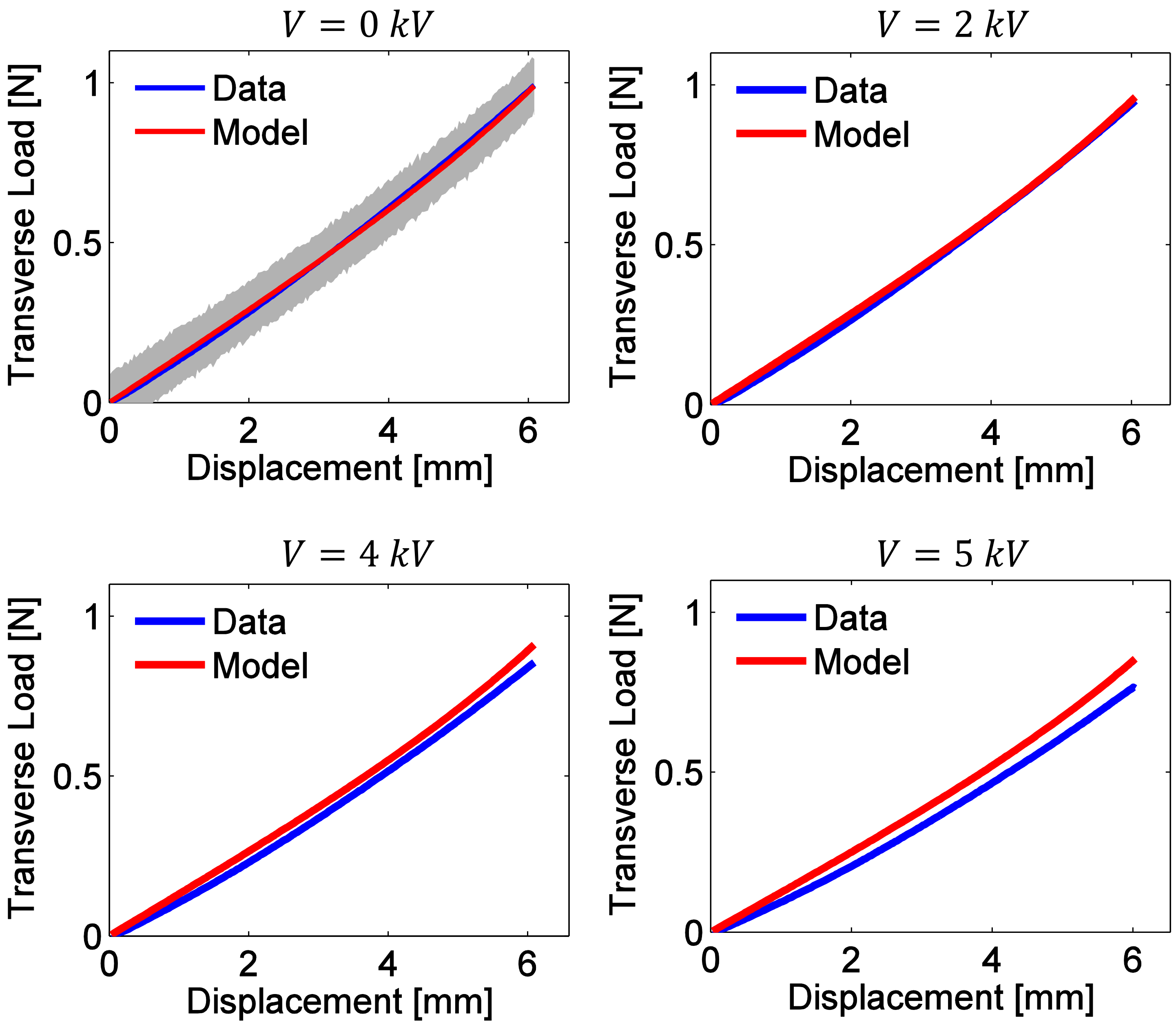
- Distribution developed from sampled parameters.
  - Posterior densities shown for $[\kappa_r, \tau]$.
- Chain panels
  - Both parameters "burned-in"



Results (2)

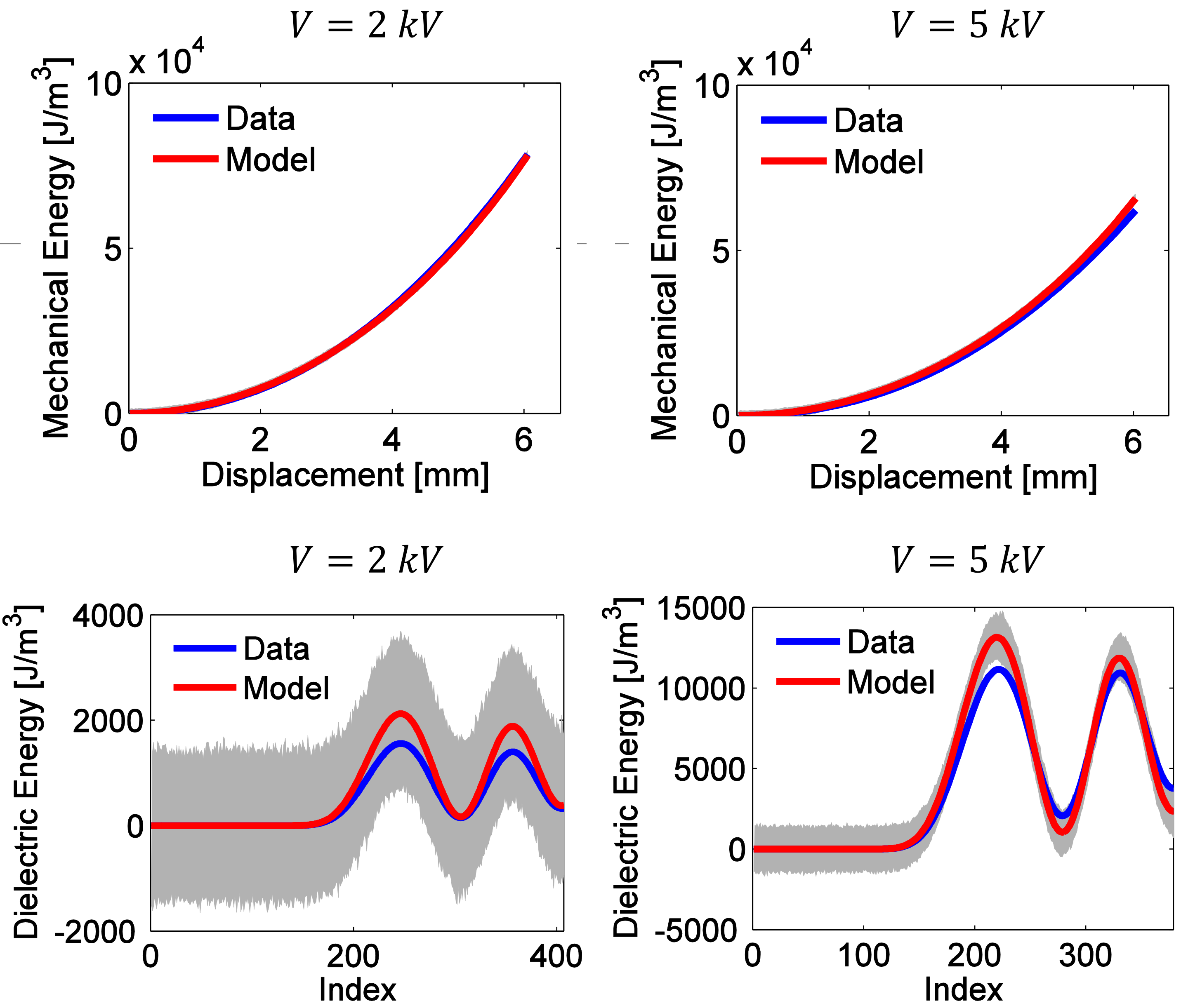
- Transverse load
  - Used average κ_r from first calibration
  - Under-predicting the effects of electrostriction





Results (3)

- Energy Formulation
  - Used both experimental data sets
  - Simultaneous model calibration
  - Uncertainty in dielectric model increased
  - Transverse load model improved.



Model Calibration

- Calibrated mean model parameters shown for three cases:
  1) Used and electric displacement data: $\theta = [\kappa_r, \tau]$
     - Previous studies: κ_r between 2.6 and 4.7
  2) Used transverse load data with no applied voltage: $\theta = [G_c, G_e, \lambda_{max}]$
  3) Used transverse load data and electric displacement data: $\theta = [G_c, G_e, \lambda_{max}, \kappa_r, \tau]$
     - Model reformulated to calculate energy to ensure common units.

Analysis	Units	(1)	(2)	(3)
G_c	kPa	-	37.3	36.0
G_e	kPa	-	6.82	0.51
λ_{max}	-	-	4.04	3.69
κ_r	-	4.30	-	5.51
τ	s	0.013	-	0.008

1. Wissler, Michael, and Edoardo Mazza. "Electromechanical coupling in dielectric elastomer actuators." Sensors and Actuators A: Physical 138.2 (2007): 384-393.

Conclusions

- Experimentally characterized
  - Transverse loading
  - Dielectric response
- Simulated response and analyzed uncertainty
- Applied different techniques for model calibration

- Future Work
  - Inhomogeneous structural model[1]
  - Deformation-dependent permittivity
  - Assess appropriateness of data fusion: scaling and sensitivity

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Acknowledgements

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  - Florida State University

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