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# Python-Based Metropolis Algorithms to Infer the Location and Intensity of an Urban Radiation Source


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## Objectives

- Implement Python-based Metropolis algorithms to support Bayesian inference for urban radiation source detection and localization.
- Sampling methods:
  - Metropolis (MH)
  - Adaptive Metropolis (AM)
  - Delayed Rejection (DR)
  - Delayed Rejection Adaptive Metropolis (DRAM)
- Adaptation of Marko Laine's MATLAB toolbox, `mcmcstat`.

## Statistical Model

- Statistical model:

$$\Upsilon_i = f_i(Q) + \varepsilon_i \quad , \quad i = 1, \ldots, n \quad , \quad \varepsilon_i \text{ iid}$$

where $\Upsilon_i, \varepsilon_i$, and $Q$ are random variables representing measurements, measurement errors, and parameters with realizations $v_i, \epsilon_i$, and $q$. Parameter dependent model is denoted by $f_i(Q)$.

- Likelihood Function:

$$\pi(v|q) = \frac{1}{(2\pi\sigma^2)^{n/2}} e^{-SS_q/2\sigma^2}$$

where $SS_q = \sum_{i=1}^{n}[v_i - f_i(q)]^2$ is the sum of squares error.

- Prediction Intervals: Sample from $\rho_Q(q)$ and $\rho_\varepsilon(\epsilon)$ and propagate through model.

## Basic Metropolis Algorithm

1. Choose initial parameter value and specify prior $\pi_0(q)$
2. Construct covariance estimate $V$ and Cholesky decomposition $R = chol(V)$.
3. For $k = 1, \ldots, M$ (# of MCMC simulations)

a. Construct candidate: $q^* = q^{k-1} + Rz$, where $z \sim N(0,1)$.

b. Compute the ratio:

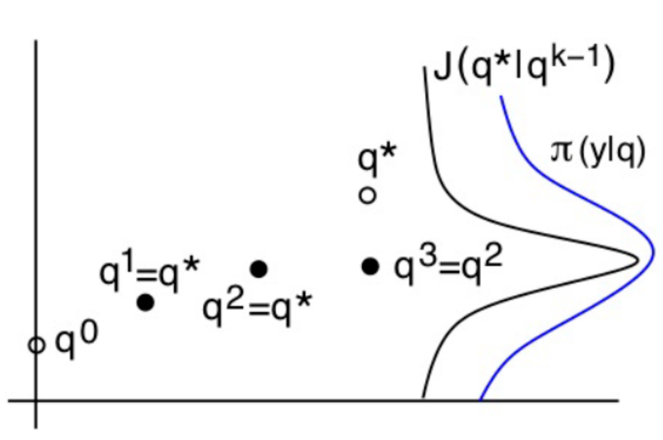
$$r(q^*|q^{k-1}) = \frac{\pi(q^*|v)}{\pi(q^{k-1}|v)} = \frac{\pi(v|q^*)\pi_0(q^*)}{\pi(v|q^{k-1})\pi_0(q^{k-1})} \; ,$$

c. If $U(0,1) < \min(1, r)$,

set $q^k = q^*$

else

set $q^k = q^{k-1}$.




## Urban Radiation Source Localization

- Model: Photon counts at a detector site given a source location and intensity:

$$\Gamma_i = \frac{\Delta t_i \epsilon_i A_i I}{4\pi|r_i - r_0|^2} \exp\left(-\sum_{n_i=1}^{N_i} \sigma_{n_i} s_{n_i}\right)$$

- Here $f_i(Q) = \Gamma_i$, where $Q = [x_0, y_0, I]$; i.e., source location and intensity.
- For more discussion of the physics for this problem, see the talk by Ralph Smith: "Mobile Sensor Network Design for Radiation Detection in an Urban Environment."

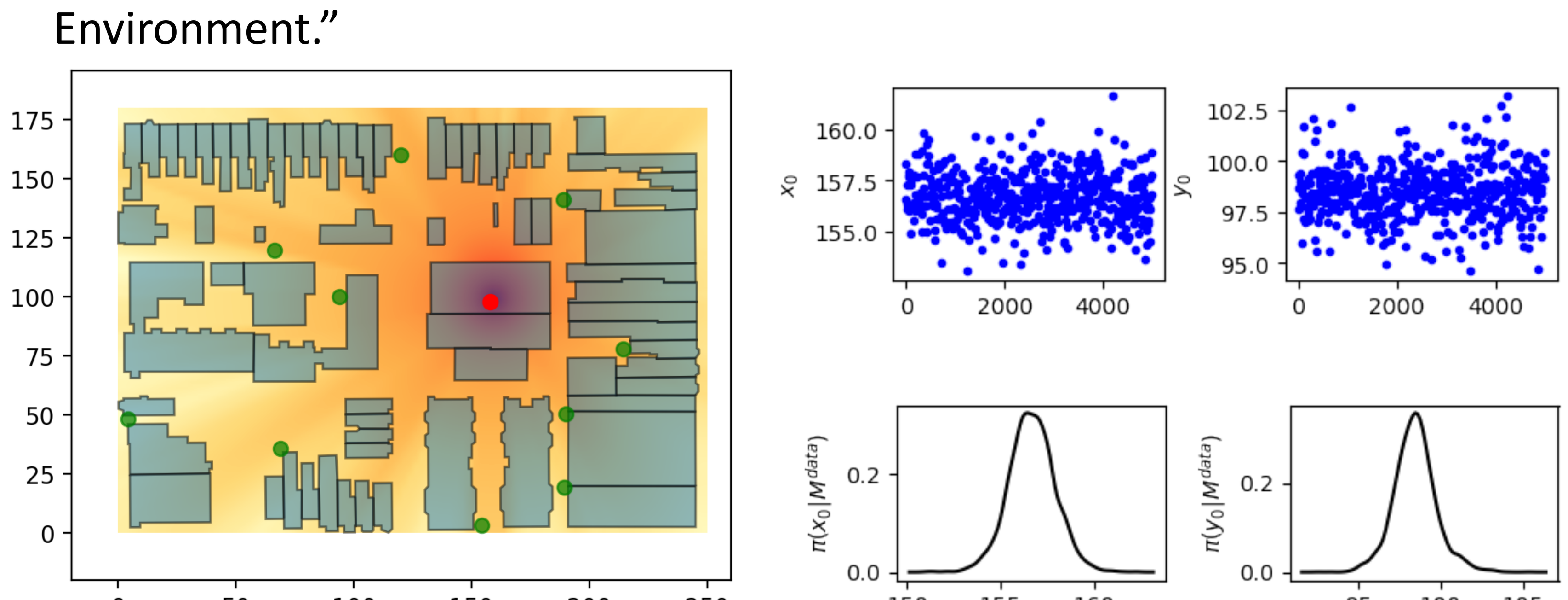
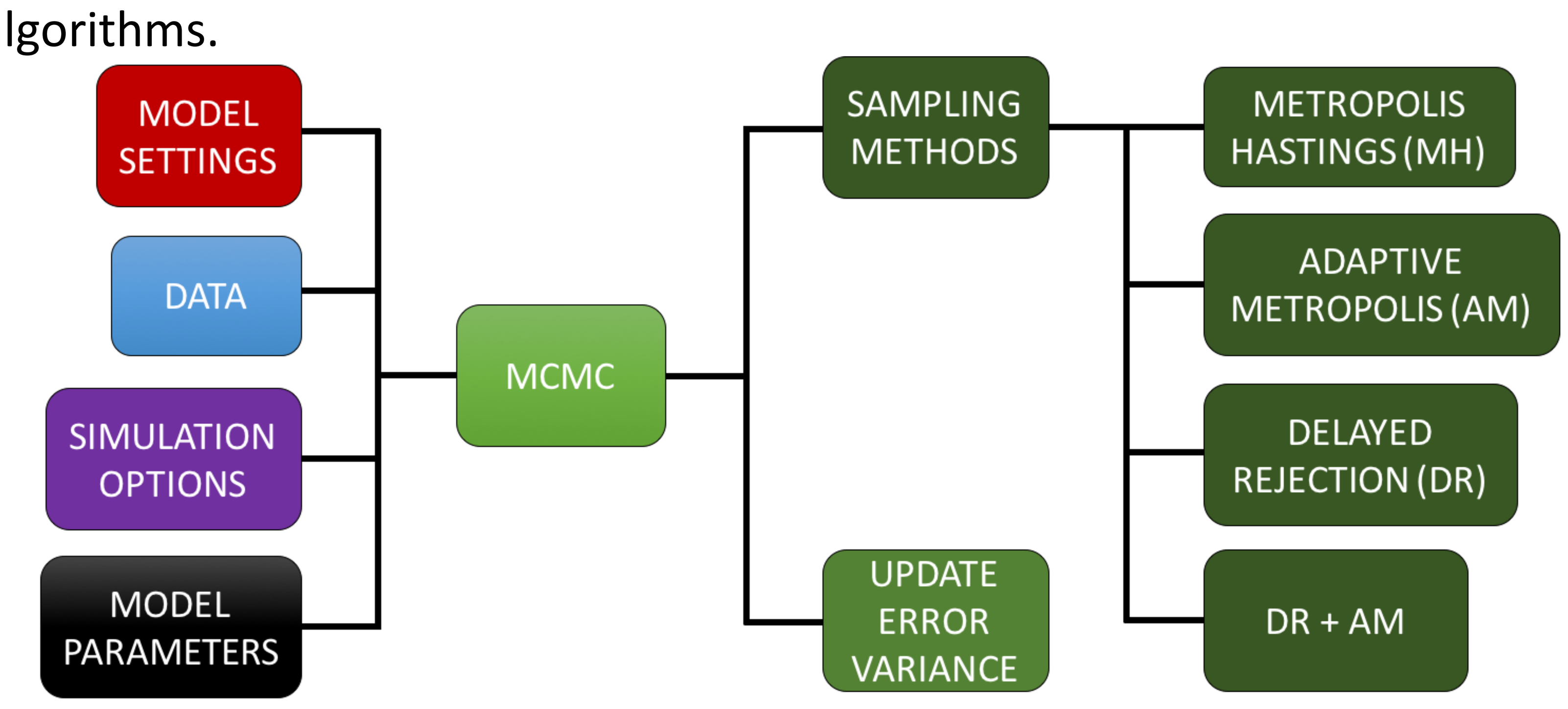


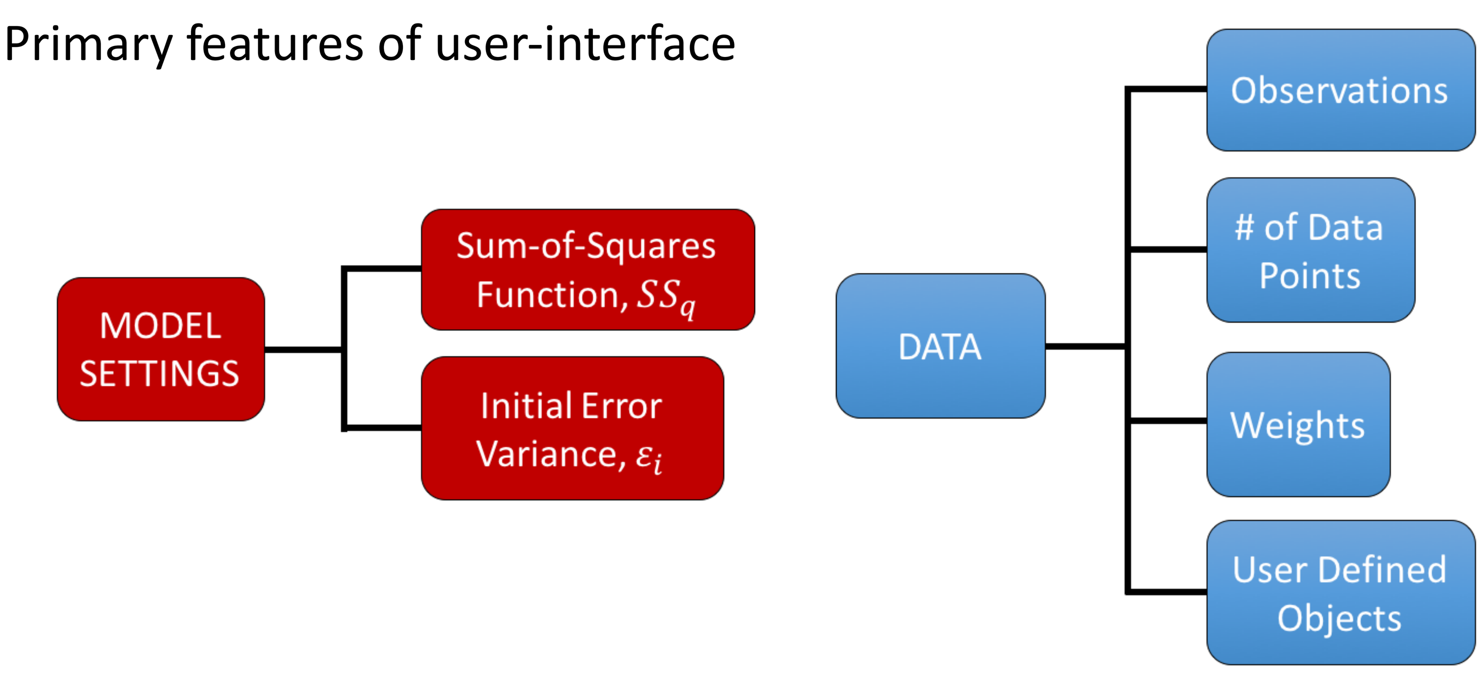

Figure: (Left) Simulated 250 m x 180 m block of downtown Washington D.C. (Right) Burned-In chains, posterior densities from parameter sampling.

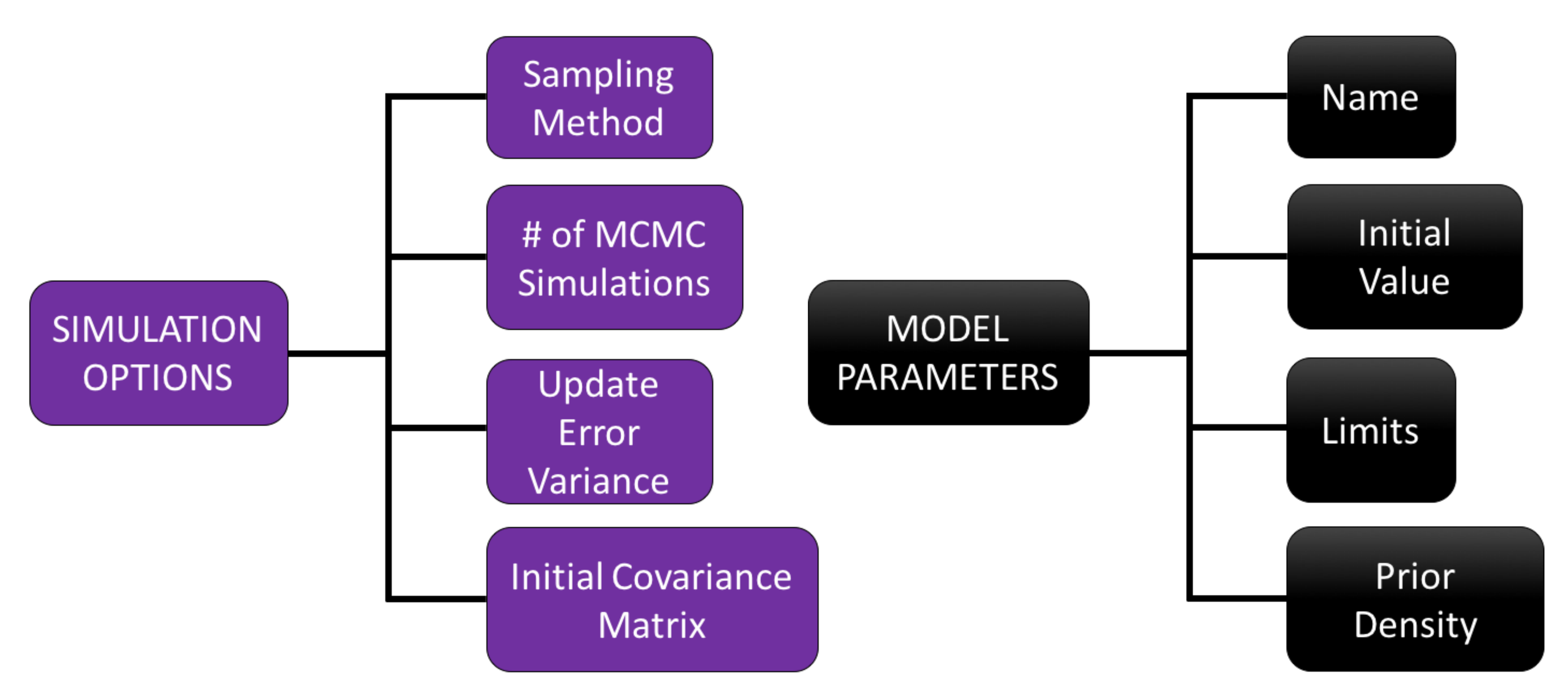
## Code Structure

- User provides calibration data, settings for statistical model and parameters, and specifies options associated with available features of the Metropolis algorithms.




- Primary features of user-interface




## Conclusions & Future Work

- Python-based algorithms return comparable results to the MATLAB toolbox at significantly reduced computational costs for Python-based models.
- Update support documents and upload demo files to website: https://prmiles.wordpress.ncsu.edu/research/

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NNSA National Nuclear Security Administration