Viscoelastic Modeling Guided by Uncertainty Quantification

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Outline

- Motivation
- Experimental setup & results
- Model theory
- Model development
  - Linear vs. nonlinear viscoelasticity
  - Bayesian uncertainty analysis
- Conclusions
Motivation

- **Application of VHB 4910 (made by 3M)**
  - Construction, robotics, flow control
- **Damping:**
  - Shock absorption via materials
  - Characterization required to develop dynamic tunability

Figure: iSprawl robotic platform (from Newton, J., “Design and Characterization of a Dielectric Elastomer Based Variable Stiffness Mechanism for Implementation onto a Dynamic Running Robot,” (2014), Figure 2.11 and Figure 4.5.)

Motivation

Two important questions:

◦ What is the most appropriate hyperelastic model?
  ◦ Ogden, Mooney-Rivlen, Langevin, Nonaffine, etc.

◦ How does hyperelasticity couple with viscoelasticity?
  ◦ Thermodynamic perspective
  ◦ Linear vs. nonlinear viscoelasticity
Experimental Setup

- **Tests:**
  - Performed at different stretch rates
  - Specimens taken through a stretch/relaxation cycling to remove viscoplastic effects (explained on next slide).

- **Stretch rate:**
  - Where
    - $\dot{\lambda}$ is the stretch rate (Hz)
    - $\dot{x}$ is the speed of the moving clamp head (mm/s)
    - $L_0$ is the initial length of the VHB specimen (mm)

Figure: MTS tensile testing of VHB 4910 (from Morton, J., “Control of the Stiffness of Robotic Appendages Using Dielectric Elastomers,” Masters Thesis, 2012)
Cyclic Loading of VHB 4910
12th Cycle for All Stretch Rates

![Graph showing nominal stress vs. strain for different stretch rates. Various curves represent different stretch rates: 6.7x10^-5 Hz, 0.047 Hz, 0.10 Hz, 0.335 Hz, 0.50 Hz, and 0.67 Hz. The y-axis represents nominal stress in kPa, and the x-axis represents the stretch ratio (λ).]
Model Theory

- **1\textsuperscript{st} Law of Thermodynamics**

\[ \rho^0 \dot{\psi} = s_{iK} \dot{F}_{iK} + \rho^0 r - Q_{I,I} - \rho^0 \Theta \dot{S} - \rho^0 S \dot{\Theta} \]

- **2\textsuperscript{nd} Law of Thermodynamics**

\[ \rho^0 \dot{S} \geq \frac{\rho^0 r}{\Theta} - \frac{1}{\rho^0} \left( \frac{Q_I}{\Theta} \right)_{,I} \]

\[ - \frac{\partial \hat{\psi}}{\partial \Gamma^\alpha_{iK}} \dot{\Gamma}^\alpha_{iK} - \frac{Q_I \Theta_{,I}}{\Theta^2} \geq 0 \]

Entropy generation

\[ - \frac{\partial \hat{\psi}}{\partial \Gamma^\alpha_{iK}} \dot{\Gamma}^\alpha_{iK} = Q^\alpha_{iK} \Gamma^\alpha_{iK} = F(\dot{\Gamma}^\alpha_{iK}, F_{iK}) \geq 0 \]

Viscoelastic stress
Model Theory

Hyperelastic: Ogden

\[ \psi_\infty^O = \sum_{p=1}^{3} \frac{\mu_p}{\alpha_p} \left( I_1^{\alpha_p} - 3 \right) \]

Hyperelastic: Nonaffine

\[ \psi_\infty^N = \frac{1}{6} G_c I_1 - G_c \lambda_{max}^2 \ln (3 \lambda_{max}^2 - I_1) + G_e \sum_j (\lambda_j + \frac{1}{\lambda_j}) \]

Viscoelastic: Linear (Spring-Dashpot)

\[ \gamma_L = \sum_\alpha \left[ \frac{1}{2} \gamma^\alpha (F_{iK} - \Gamma_{iK}^\alpha) (F_{iK} - \Gamma_{iK}^\alpha) \right] \]

Viscoelastic: Nonlinear

\[ \gamma_{NL} = \sum_\alpha \left[ \frac{1}{2} \gamma^\alpha \Gamma_{iK}^\alpha \Gamma_{iK}^\alpha - \beta^\alpha \frac{\partial \psi_\infty}{\partial F_{iK}} \Gamma_{iK}^\alpha + \beta^\alpha \psi_\infty \right] \]


Bayesian Uncertainty Analysis

- Optimization performed using Markov Chain Monte Carlo algorithm (MCMC).
  - Random sampling
  - Global optimization
  - Parameter Relationships

- Bayes’ Relation
  - \( \pi(\theta | y) = \frac{p(y|\theta)p_0(\theta)}{\int_{\mathbb{R}^p} p(y|\theta)p_0(\theta)d\theta} \)
Results

- **Hyperelastic:**
  - Ogden
  - Open

- **Viscoelastic:**
  - Linear

- Results shown for 4 stretch rates
Results

• **Hyperelastic:**
  • Ogden
  • Fixed

• **Viscoelastic:**
  • Linear

• Results shown for 4 stretch rates
Results

- **Hyperelastic:**
  - Nonaffine
  - Fixed

- **Viscoelastic:**
  - Nonlinear

Results shown for 4 stretch rates
<table>
<thead>
<tr>
<th>Hyperelastic</th>
<th>Ogden</th>
<th>Nonaffine</th>
<th>Ogden</th>
<th>Nonaffine</th>
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</thead>
<tbody>
<tr>
<td>Viscoelastic</td>
<td>Linear</td>
<td>Linear Nonlinear</td>
<td>Linear</td>
<td>Linear Nonlinear</td>
</tr>
<tr>
<td>Optimization</td>
<td>Open</td>
<td>Fixed</td>
<td>All error is calculated in kPa².</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stretch Rate</th>
<th>Ogden</th>
<th>Linear Nonlinear</th>
<th>Ogden</th>
<th>Linear Nonlinear</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.7x10⁻⁵ Hz</td>
<td>0.50</td>
<td>0.87 1.86</td>
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<td>0.87 1.86</td>
</tr>
<tr>
<td>0.0472 Hz</td>
<td>0.60</td>
<td>0.68 2.69</td>
<td>9.34</td>
<td>11.0 2.71</td>
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<tr>
<td>0.10 Hz</td>
<td>0.67</td>
<td>0.81 2.96</td>
<td>14.6</td>
<td>16.1 3.03</td>
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<tr>
<td>0.335 Hz</td>
<td>3.50</td>
<td>4.05 14.4</td>
<td>62.9</td>
<td>67.8 17.2</td>
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<tr>
<td>0.50 Hz</td>
<td>4.40</td>
<td>5.70 14.6</td>
<td>54.4</td>
<td>59.2 33.0</td>
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<tr>
<td>0.67 Hz</td>
<td>4.80</td>
<td>6.42 22.5</td>
<td>82.1</td>
<td>86.6 25.7</td>
</tr>
</tbody>
</table>

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Parameter Distributions

- Distribution developed from sampled parameters.
- Example distribution results:
  - Nonaffine, nonlinear
  - Fixed hyperelastic parameters
  - Fastest stretch rate (0.67 Hz)
Conclusions and Future Work

- **Conclusions:**
  - Quantified stretch rate independent hyperelastic parameters
  - Coupled hyperelasticity with linear and nonlinear viscoelasticity
  - Nonlinear viscoelastic model ~3X more accurate for VHB4910

- **Future:**
  - Model validation over stretch rates not tested
  - Nonlinear finite element implementation for complex loading
  - Incorporate effects of viscoplasticity
  - Multiscale statistical analysis of polymer networks

![Power law relation on time constant](image)
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